Mixing parametrizations for ocean climate modelling

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Motivation

- Turbulent processes are a crucial factor for three-dimensional ocean general circulation models. They are not described by the equations of the general circulation model, so they have to be parameterized.
- Air–sea heat fluxes in the ocean circulation model are calculated using the model sea surface temperature, which depends on turbulent mixing. Thus, the specification of boundary conditions also depends to a large extent on which type of mixing parameterization is used.
- Various dependences of vertical exchange coefficients on stratification and velocity shear or local Richardson number are often used. Turbulence models of different levels of the consideration of physical factors are also used for this purpose. It is pointed out that the use such turbulence models gives the better results, but increases the computational time.
- **It is therefore important to apply a sufficiently complete physical formulation of a turbulence model and design a computationally efficient algorithm of such a model.**

Institute of Numerical Mathematics Ocean Model (INMOM)

INMOM is a three-dimensional, free surface, sigma coordinate circulation model. The hydrostatic and Boussinesq approximations are used by the model. The model variables are horizontal velocity components, potential temperature, and salinity as well as free surface height.

$$\sigma = \frac{z - \zeta}{H - \zeta},$$

where H-ζ is the full depth, ζ is the free surface height and z is geopotential vertical coordinate.
The model uses spatial approximations on a staggered C-grid. The splitting method allows to implement efficient implicit time-integration schemes for the transport and diffusion equations (the Crank-Nicholson approximation is used for transport processes, and an implicit scheme is used for calculating diffusion and viscosity).

Model variant for present experiments

The present version of the model is realized for coupled Atlantic (open boundary at 30°S, including Mediterranean) - Arctic Ocean – Bering Sea region

A rotation of the model grid is employed to avoid the problem of converging meridians over the Arctic ocean. The model North Pole is located at geographical equator, 120°W.

1/4° horizontal eddy-permitting resolution is used (620x440 grid points) and 40 unevenly spaced vertical levels.

Biharmonic operator is used for lateral viscosity.

The EVP (elastic- viscous- plastic) dynamic - thermodynamic sea ice model (Iakovlev 2005, Hunke and Dukowicz 1997) is embedded.

Parameterizations for vertical diffusion and viscosity are used in 6 variants.

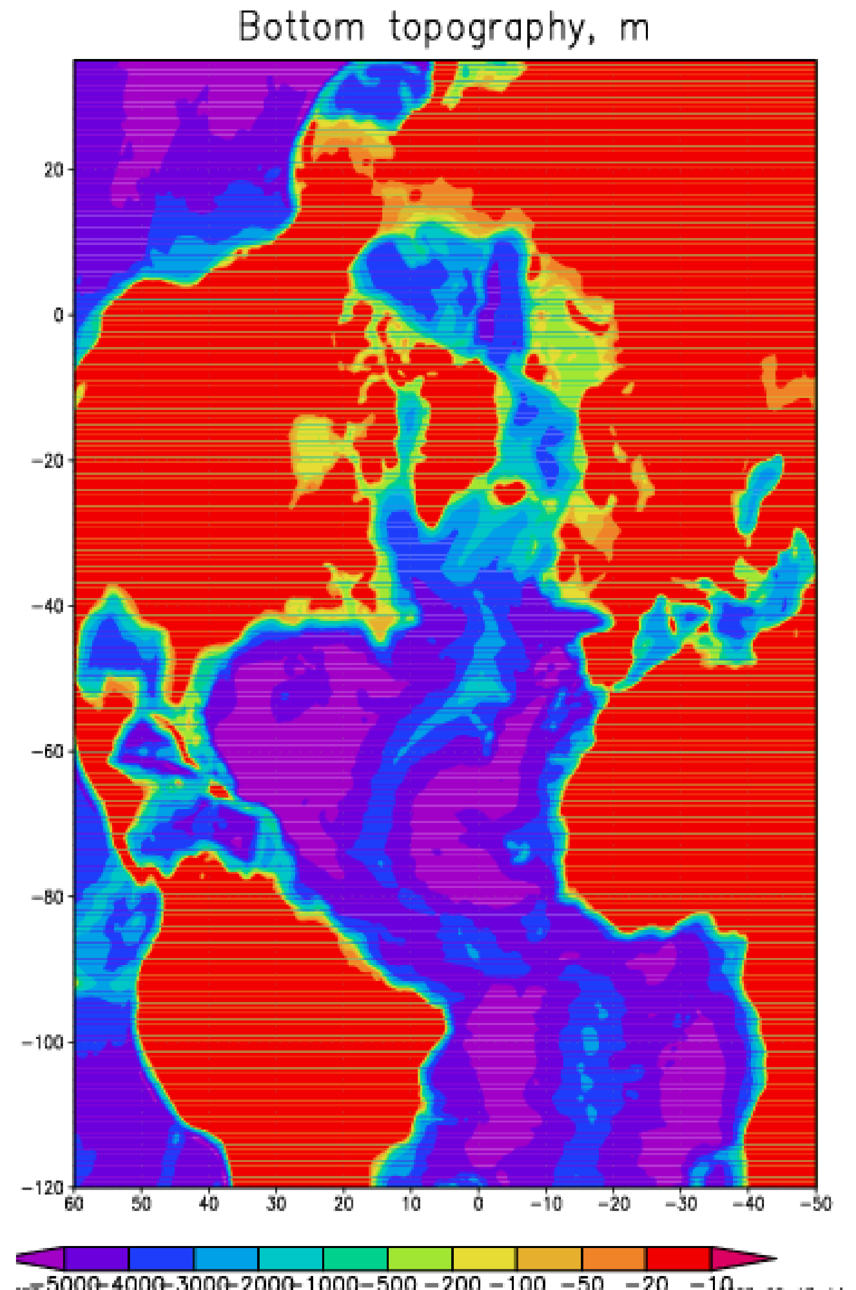
Time steps:

1 hour –circulation;

5 minutes – splitting equations for turbulence characteristics.

CORE-2 data for boundary conditions (1948-2009)

Model domain



Vertical fluxes parameterizations:

$$\overline{u'(v')w'} = -\nu_u \frac{1}{H}\frac{\partial u(\mathrm{v})}{\partial \sigma}, \qquad \overline{T'(S')w'} = -\nu_{T(S)} \frac{1}{H}\frac{\partial T(S)}{\partial \sigma}$$

$$\nu_u = \frac{C_S^U}{c_S^0}\cdot\frac{k}{\omega}, \qquad \nu_{T(S)} = \frac{C_S^U}{\mathrm{Pr}\cdot c_S^0}\cdot\frac{k}{\omega}$$

$k = \overline{(u')^2 + (v')^2 + (w')^2}$ - Turbulence Kinetic Energy (TKE), ([k]=см^2/s^2)

$\omega = \dfrac{\varepsilon}{(c_S^0)^4 \cdot k}$ - a frequency characteristic of the turbulence decay process

ε - Dissipation rate of TKE (см^2/s^3), C_S^U, c_S^0 - Stability functions, Pr - Prandtl number.

The two-equation Turbulence Model :

$$\frac{dk}{dt} = \frac{1}{H^2}\cdot\frac{\partial}{\partial\sigma}\left(\frac{\nu_u}{\sigma_k}\cdot\frac{\partial k}{\partial\sigma}\right) + \nu_u\cdot G^2 - \nu_\rho\cdot N^2 - (c_S^0)^4\cdot\omega\cdot k$$

$$\frac{d\omega}{dt} = \frac{1}{H^2}\cdot\frac{\partial}{\partial\sigma}\left(\frac{\nu_u}{\sigma_\omega}\cdot\frac{\partial\omega}{\partial\sigma}\right) + \frac{\omega}{k}\cdot\left(c_1^\omega\cdot\nu_u\cdot G^2 - c_3^\omega\cdot\nu_\rho\cdot N^2 - c_2^\omega\cdot(c_S^0)^4\cdot k\cdot\omega\right)$$

|______First stage____| **|____________Second Stage ______________|**

N^2, G^2 - buoyancy and shear frequencies square extent (**Hz^2**)

(example: Warner, Sherwood, Arango, and Signell. Ocean Modelling, 2005)

splitting method for *turbulence model equations:*

All the required grid functions have been solved by circulation model at the time moment t_{j+1}. *Using the splitting method, it is now necessary* to solve a set of equations for the turbulent exchange.

First stage of splitting = transport and vertical diffusion:

$$D_t k = \frac{1}{H}\frac{\partial}{\partial\sigma}\frac{\nu_u}{\sigma_k}\frac{\partial k}{\partial\sigma} \qquad D_t\omega = \frac{1}{H}\frac{\partial}{\partial\sigma}\frac{\nu_u}{\sigma_\omega}\frac{\partial\omega}{\partial\sigma}$$

Boundary Conditions for First stage of splitting :

$$\sigma = 0: \quad \frac{\nu_u}{\sigma_k}\frac{1}{H}\frac{\partial k}{\partial\sigma} = -C_g\cdot\left(u_*^S\right)^3, \qquad u_*^S = \left(\sqrt{\tau_{ax}^2+\tau_{ay}^2}/\rho_w\right)^{1/2}$$

Second Stage of Splitting : generation – dissipation TKE

$$\frac{d\omega}{dt} = B - C\cdot\omega^2, \qquad \frac{dk}{dt} = \left(\frac{A}{\omega} - D\cdot\omega\right)\cdot k.$$

,

$$A = (c_S^0)^{-1}\cdot\left(C_S^U\cdot G^2 - (C_S^U/\mathrm{Pr})\cdot N^2\right), \qquad B = (c_S^0)^{-1}\cdot\left(c_1^\omega\cdot C_S^U\cdot G^2 - c_3^\omega\cdot(C_S^U/\mathrm{Pr})\cdot N^2\right),$$

$$C = c_2^\omega\cdot\left(c_S^0\right)^4, \qquad D = \left(c_S^0\right)^4.$$

.

,

Analytical solution for generation – dissipation stage

(with assumption: $C_S^U = c_S^0 \;\; in \;\; \Delta_k \sigma$):

$$\omega = -\sqrt{\frac{B}{C}} \cdot \frac{1+\tilde{C}\cdot\exp\left(2\sqrt{B\cdot C}\cdot t\right)}{1-\tilde{C}\cdot\exp\left(2\sqrt{B\cdot C}\cdot t\right)},$$

$$\tilde{C} = \frac{\omega^0 + \sqrt{B/C}}{\omega^0 - \sqrt{B/C}},$$

$$k = k^0 \cdot \left[\frac{\left(1+\tilde{C}\exp\left(2\sqrt{B\cdot C}\cdot t\right)\right)^2}{\left(1+\tilde{C}\right)^2\exp\left(2\sqrt{B\cdot C}\cdot t\right)}\right]^{A/(2B)} \cdot \left[\frac{\left(1-\tilde{C}\right)^2\exp\left(2\sqrt{B\cdot C}\cdot t\right)}{\left(1-\tilde{C}\exp\left(2\sqrt{B\cdot C}\cdot t\right)\right)^2}\right]^{D/(2C)}.$$

More convenient form analytical solution:

$$\omega = -r_d \cdot (r_m + r_p \cdot a_r)/(r_m - r_p \cdot a_r), \qquad k = k^0 \cdot \left(\frac{E_1}{E_2}\right)^{A/(2B)} \cdot \left(\frac{E_3}{E_4}\right)^{D/(2C)},$$

$$r_d = \sqrt{B/C},\;\; r_m = \omega^0 - r_d,\; r_p = \omega^0 + r_d,\;\; a_r = \exp\left(2\sqrt{B\cdot C}\cdot t\right),$$

$$E_1 = (r_m + r_p \cdot a_r)^2,\;\; E_2 = 4\cdot(\omega^0)^2 \cdot a_r,\;\; E_3 = 4\cdot r_d^{\,2}\cdot a_r,\; E_4 = (r_m - r_p\cdot a_r)^2.$$

Asymptotic form for the generation – dissipation stage:

$$\omega = \sqrt{\frac{B}{C}},$$

$$k = k^0 \exp(\gamma t),$$

$$\gamma = \frac{A}{\omega} - D\cdot\omega.$$

Numerical explicit - implicit scheme for generation – dissipation stage:

$$\frac{\omega^{n+1}-\omega^{n}}{\tau_t}=B-C\cdot\omega^{n}\cdot\omega^{n+1}\rightarrow\quad \omega^{n+1}=\frac{B\cdot\tau_t+\omega^{n}}{1+C\cdot\omega^{n}\cdot\tau_t},$$

$$\frac{k^{n+1}-k^{n}}{\tau_t}=\alpha\cdot k^{n}\qquad\rightarrow\qquad k^{n+1}=k^{n}\cdot(1+\alpha\cdot\tau_t),\ \alpha>0,$$

$$\frac{k^{n+1}-k^{n}}{\tau_t}=\alpha\cdot k^{n+1}\qquad\rightarrow\qquad k^{n+1}=\frac{k^{n}}{1-\alpha\cdot\tau_t},\qquad \alpha<0,$$

$$\alpha=\frac{A}{\omega^{n}}-D\cdot\omega^{n}.$$

Prandtl Number

(Blanke, B. and P. Delecluse, 1993; Madec G., 2008: NEMO ocean engine (version 3.2).

$$\mathrm{Pr} = 1, \quad N^2 < F_{cr}^2,$$

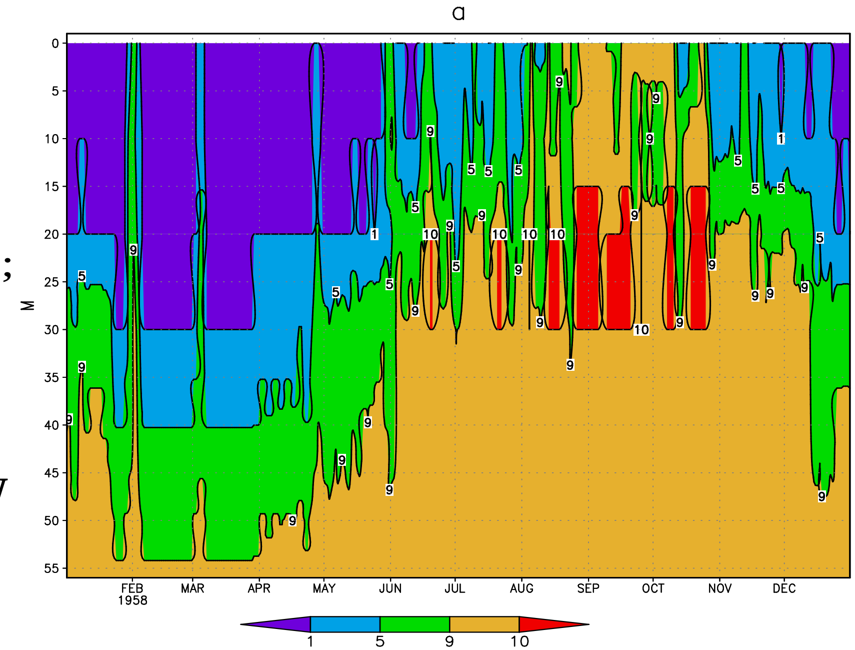
$$\mathrm{Pr} = 10, \ (N^2 \geq F_{cr}^2) + (G^2 \leq \frac{1}{2} \cdot F_{cr}^2),$$

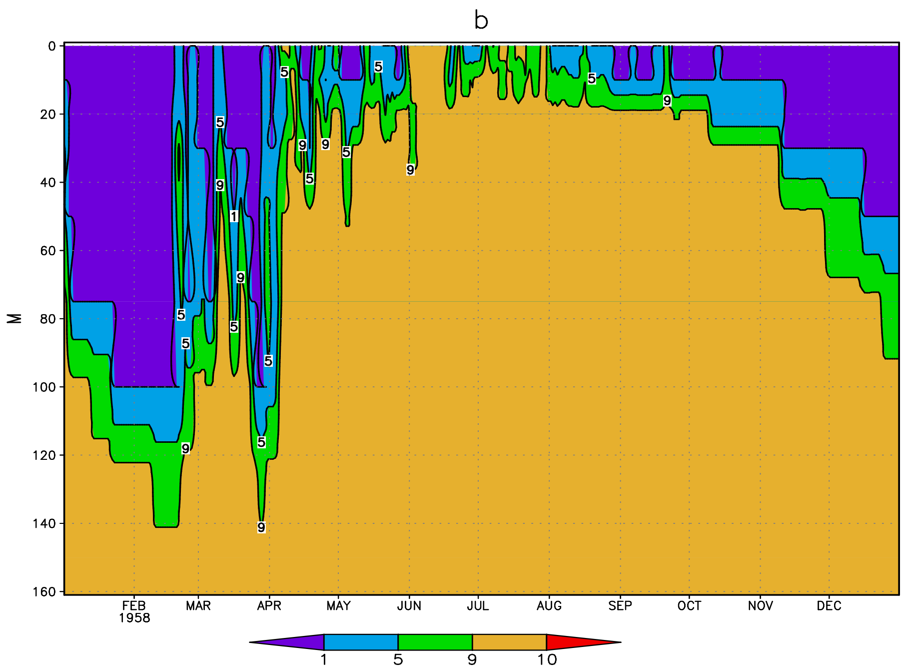
$$\mathrm{Pr} = \begin{cases} 1, \ Ri \leq 0.2 \\ 5 \cdot Ri, \ 0.2 < Ri < 2, \ (N^2 \geq F_{cr}^2) + (G^2 > \frac{1}{2} \cdot F_{cr}^2); \\ 10, \ Ri \geq 2 \end{cases}$$

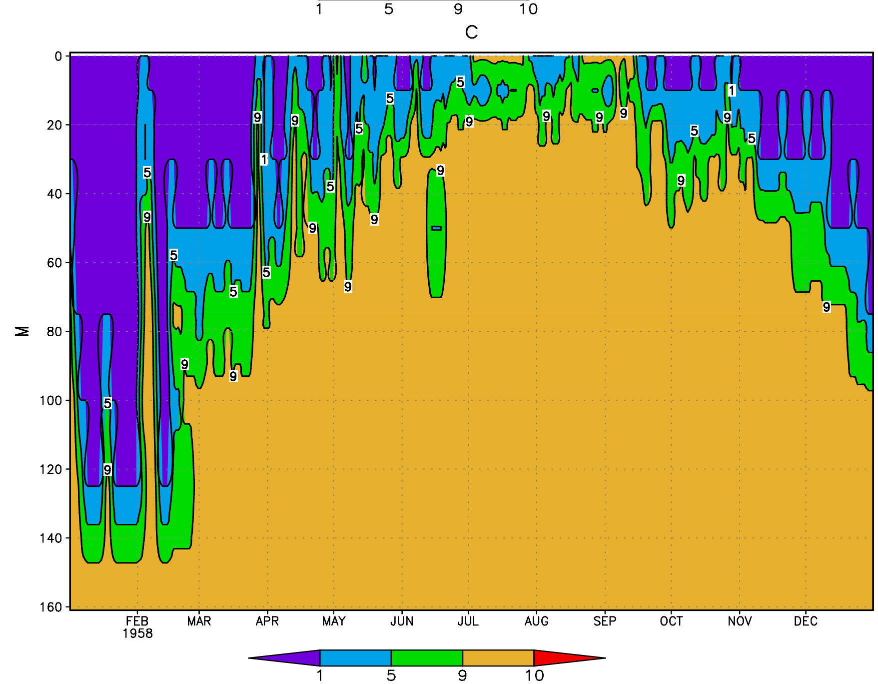
$$\mathrm{Ri} = \frac{N^2}{G^2}, \quad F_{cr}^2 = 0.5 \cdot 10^{-6}\, Hz^2. \qquad \nu_{T(S)} = \frac{\nu_u}{\mathrm{Pr}}.$$

Typical Prandtl number seasonal cycle at the 30^0W meridian in the North Atlantic:

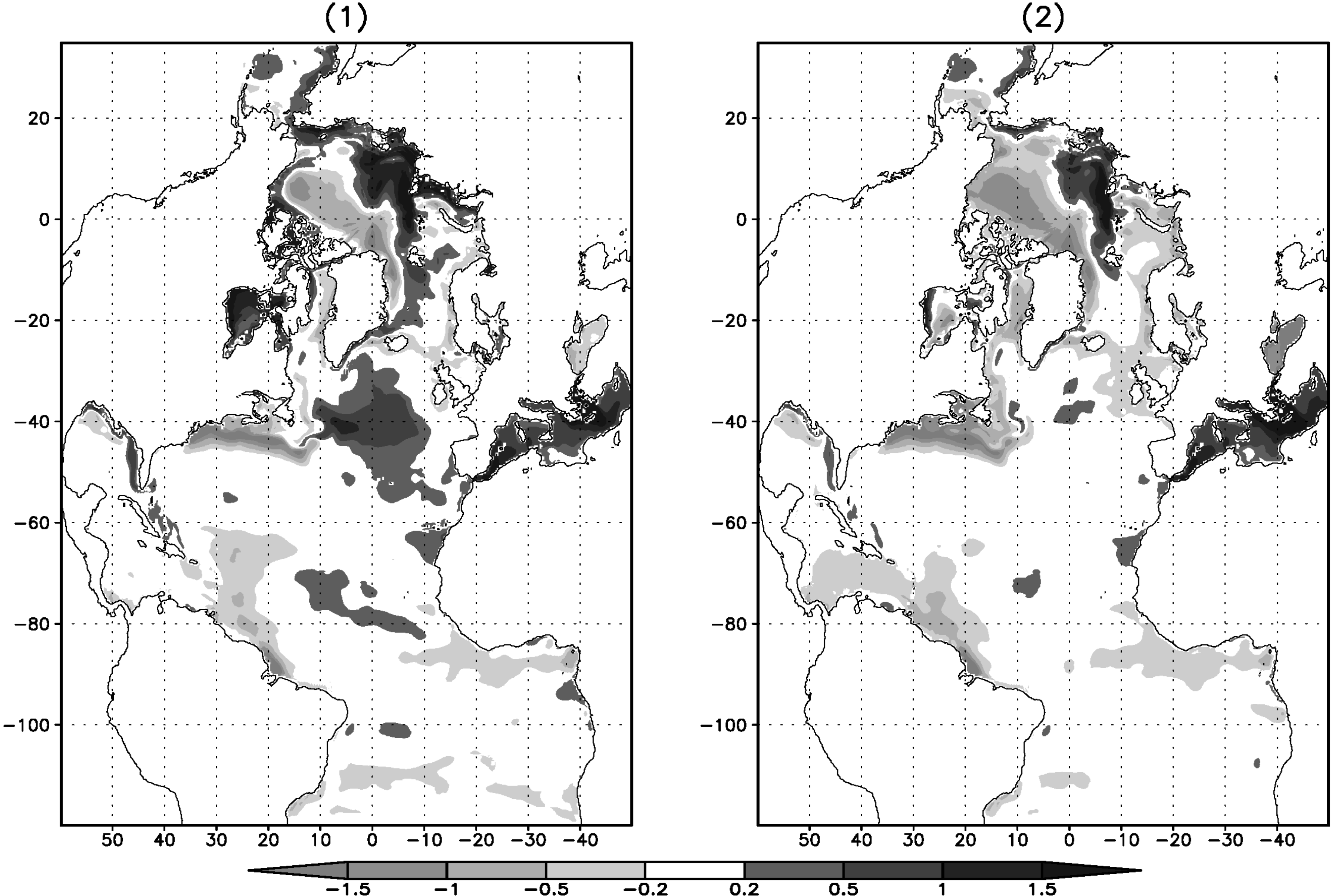
(a) - 10^0N; (b) - 30^0N; (c) - 50^0N.





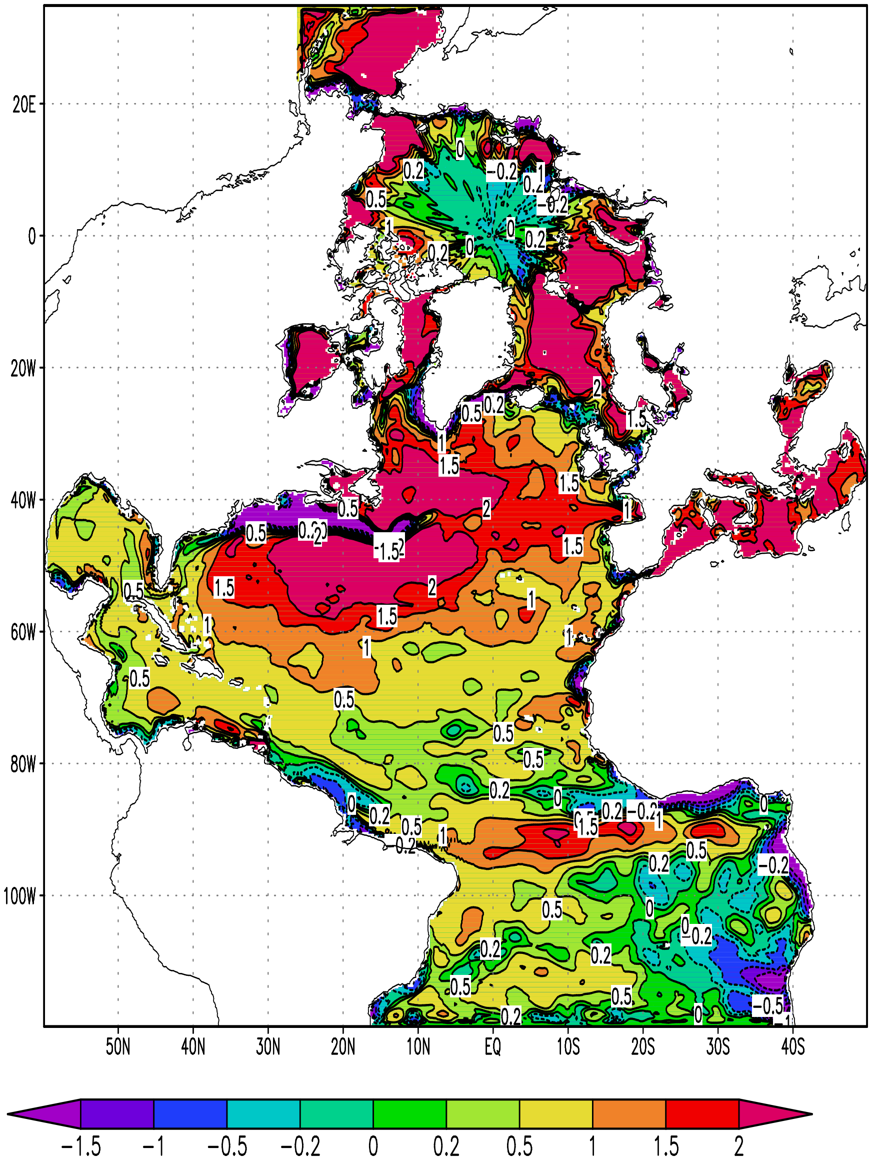


Salinity difference "climate minus model" (ppt): (1) – Pacanowski-Philander, (2) – analytical solution splitting algorithm. Layer 0-50m. 20 years integration. Model coordinates.

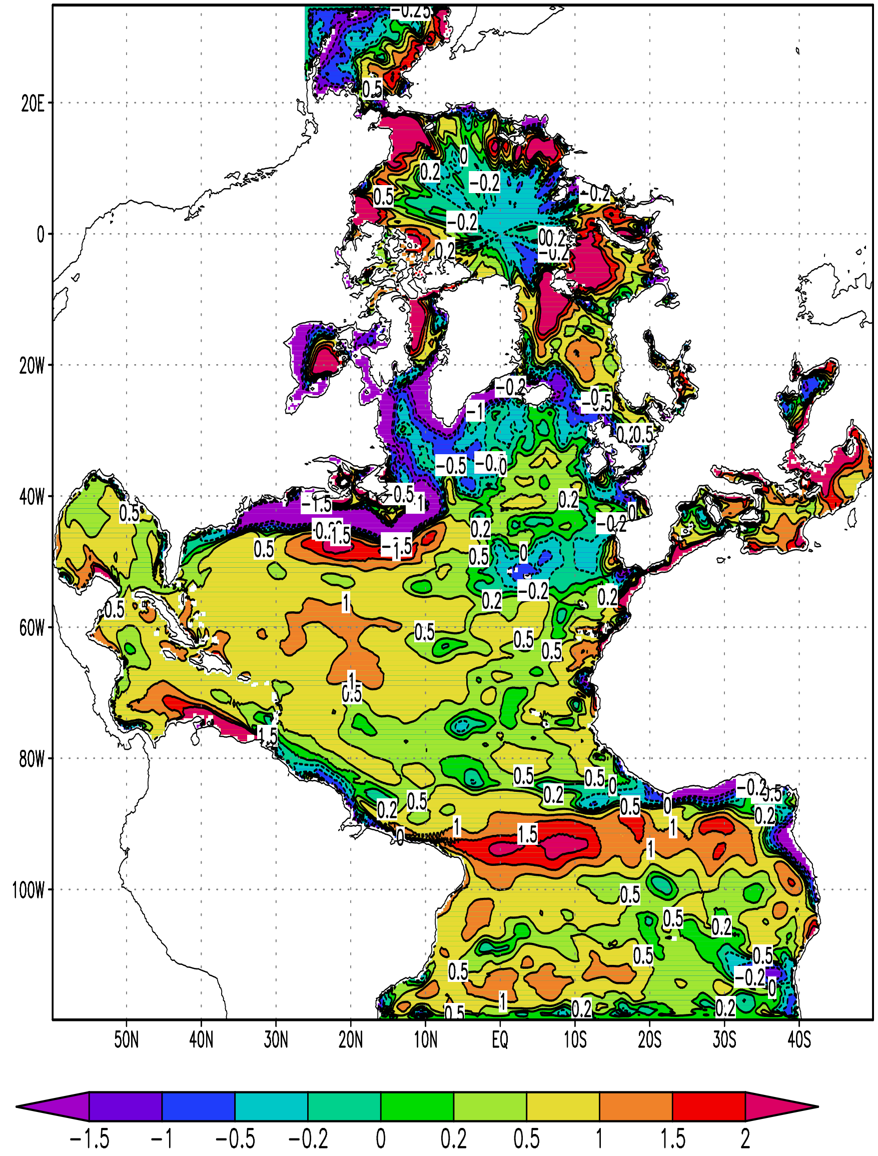


Temperature difference "climate minus model" (^{0}C): (1) – Pacanowski-Philander, (2) – analytical solution splitting algorithm. Augest. Layer 0-10m. 62 years integration. Model coordinates.

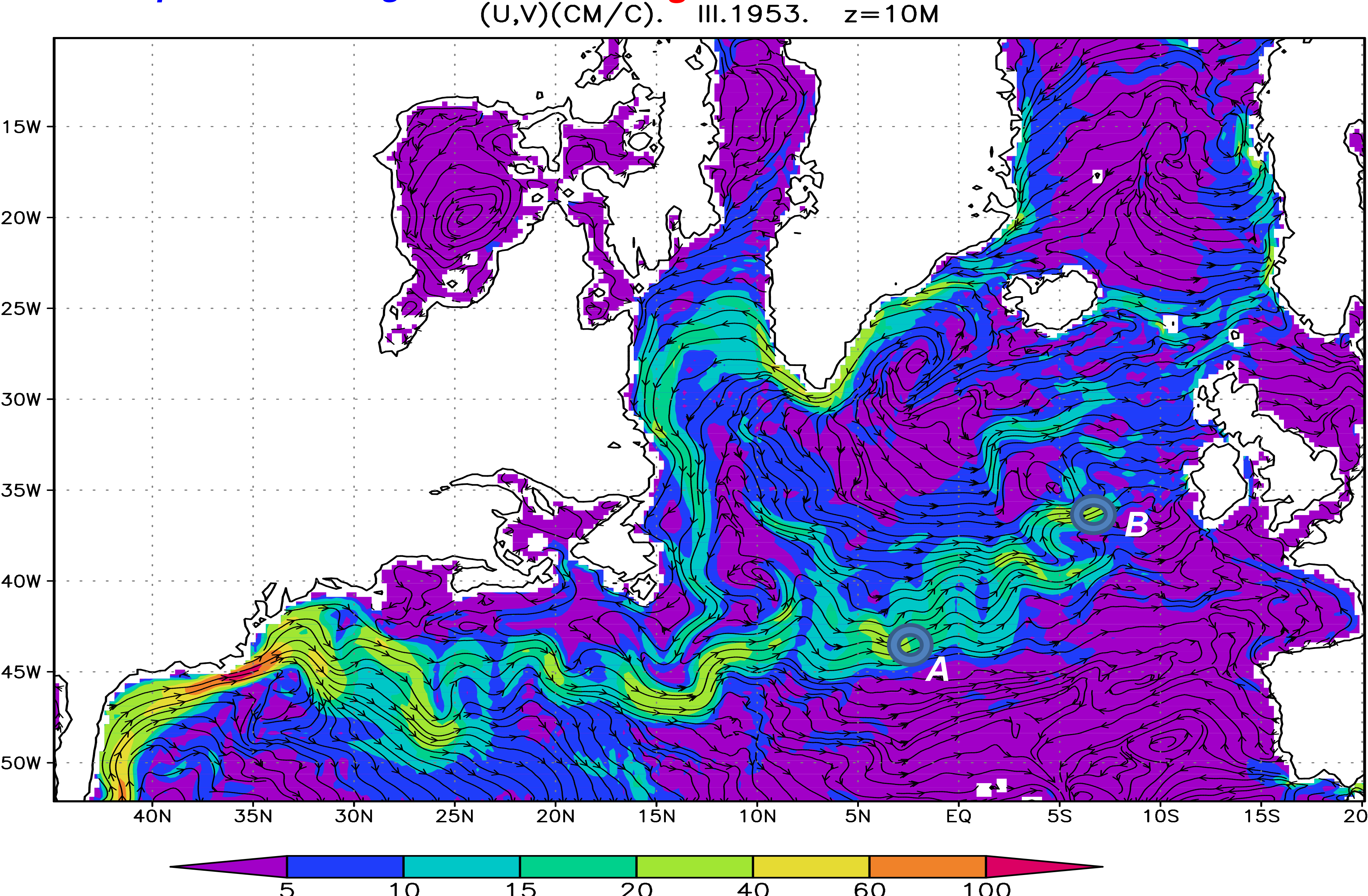
(1)



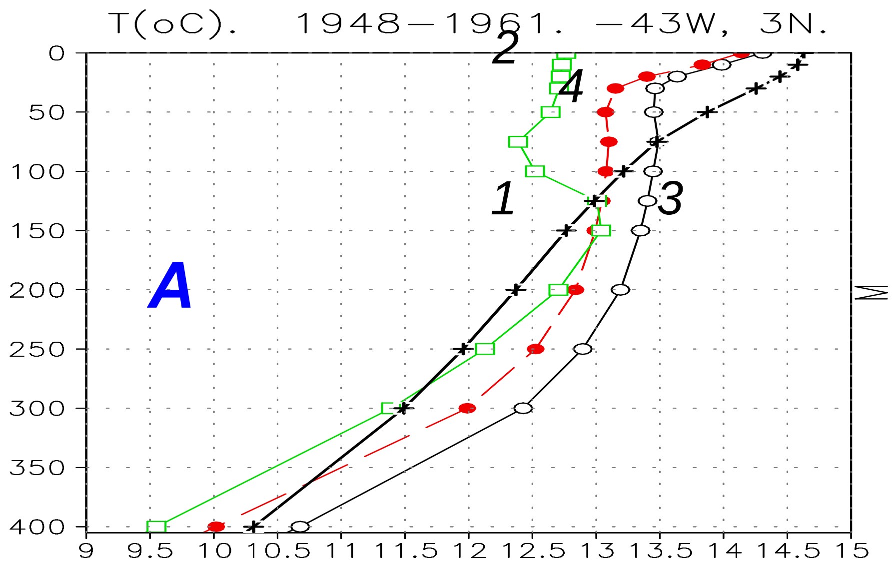
(2)

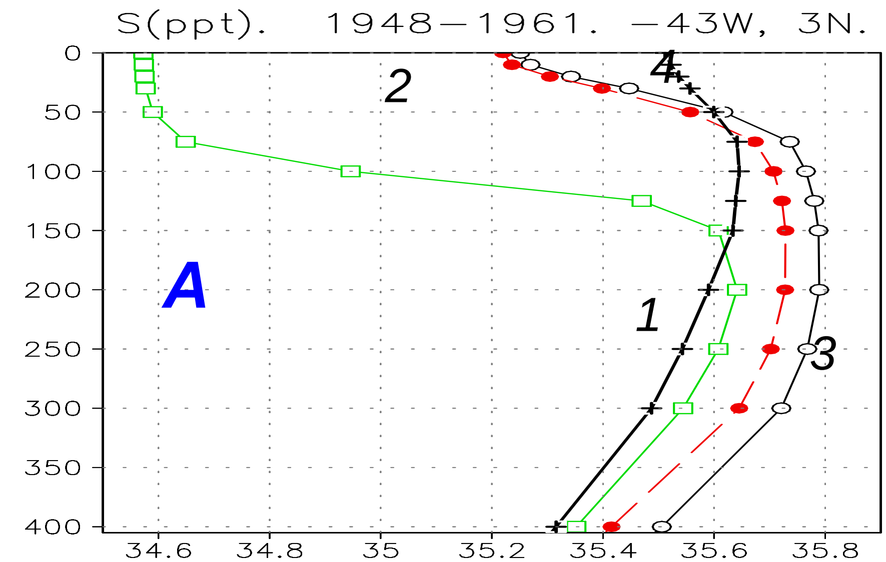


Example of the North Atlantic model circulation. Elect spherical trapezes 1X1 degree = 25 model grids. Model coordinates.

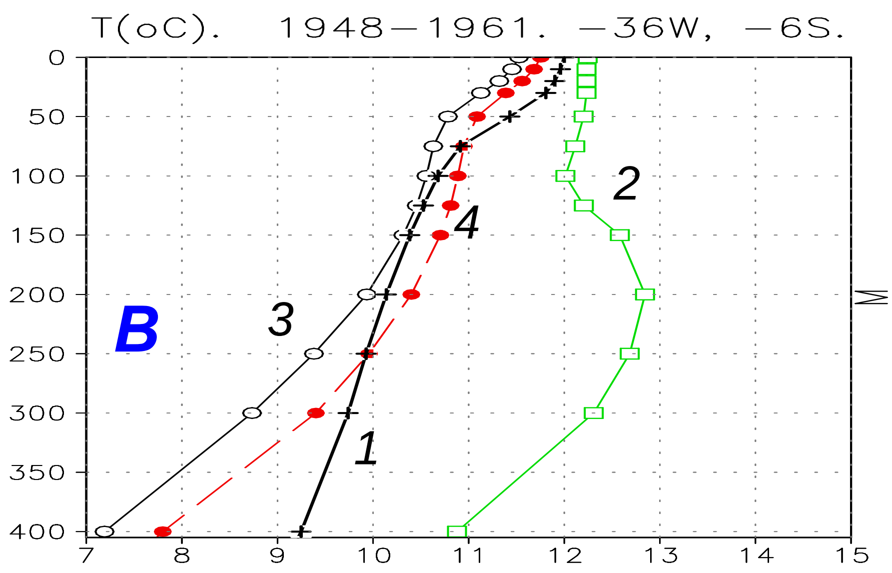


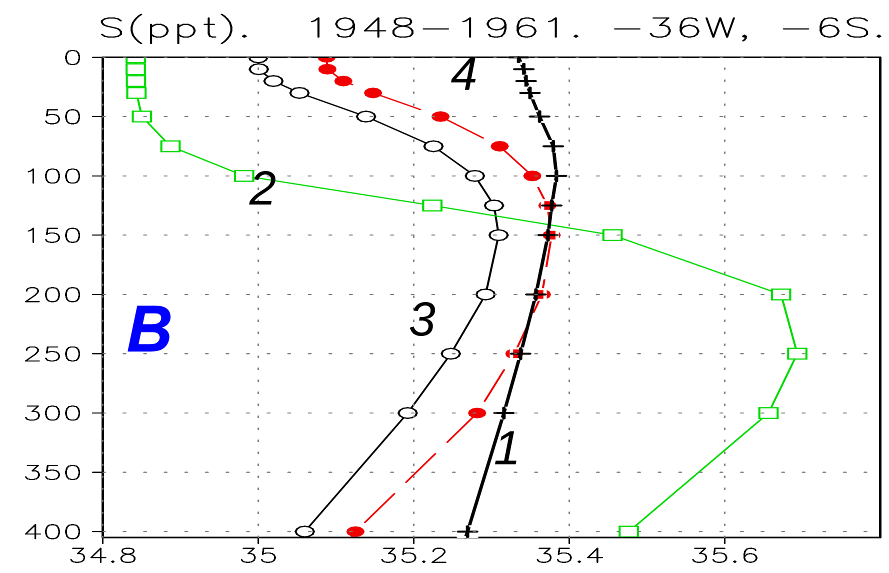
1 - Climate; **2 - Pacanowski-Philander Parameterization**;
3 – numerical (step 5 min); *4 – analytics*





(SOTA – Splitting Omega-Turbulence Algorithm)





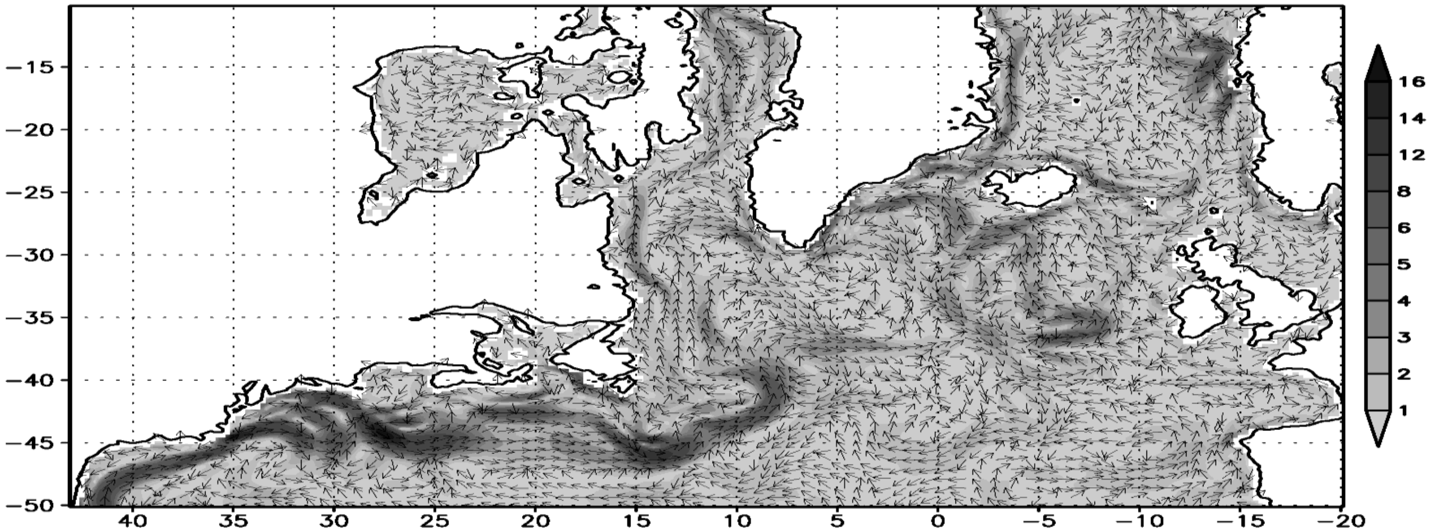
Sensitivity to mixing parameterizations.

(1) The velocity difference "analytical solution k-omega minus Pacanowski-Philander" (cm/s)

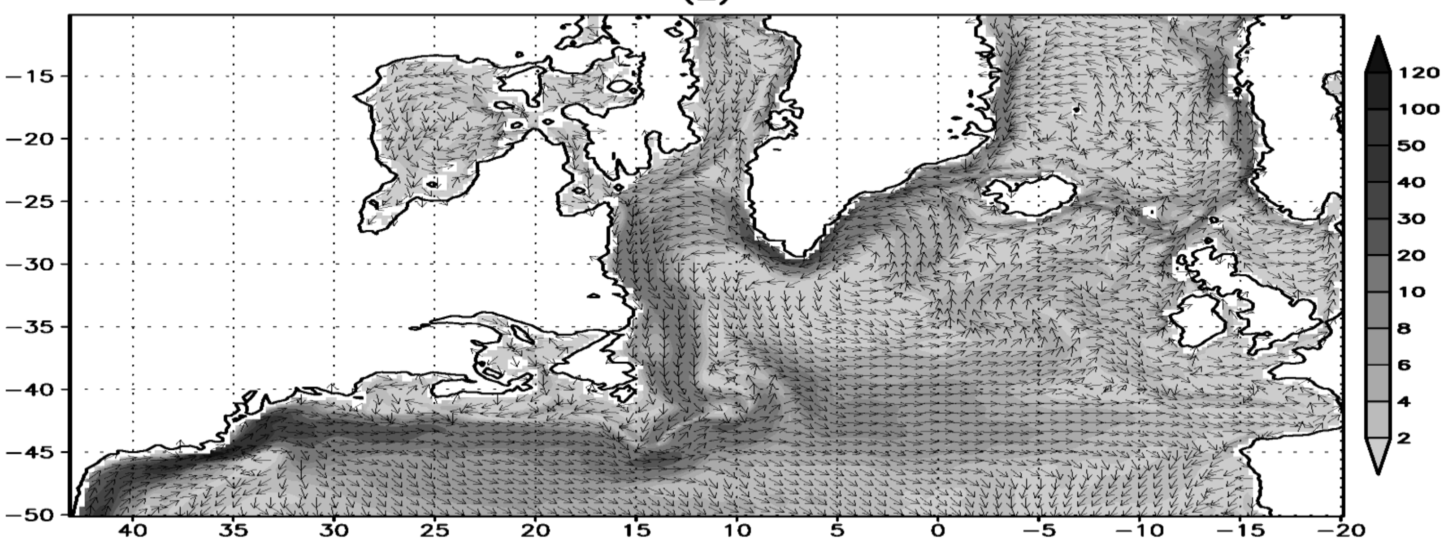
(2) Model circulation for comparison with (1)

Atlantic ocean. Layer 0-50m. Model coordinate. 20 years integration.

(1)



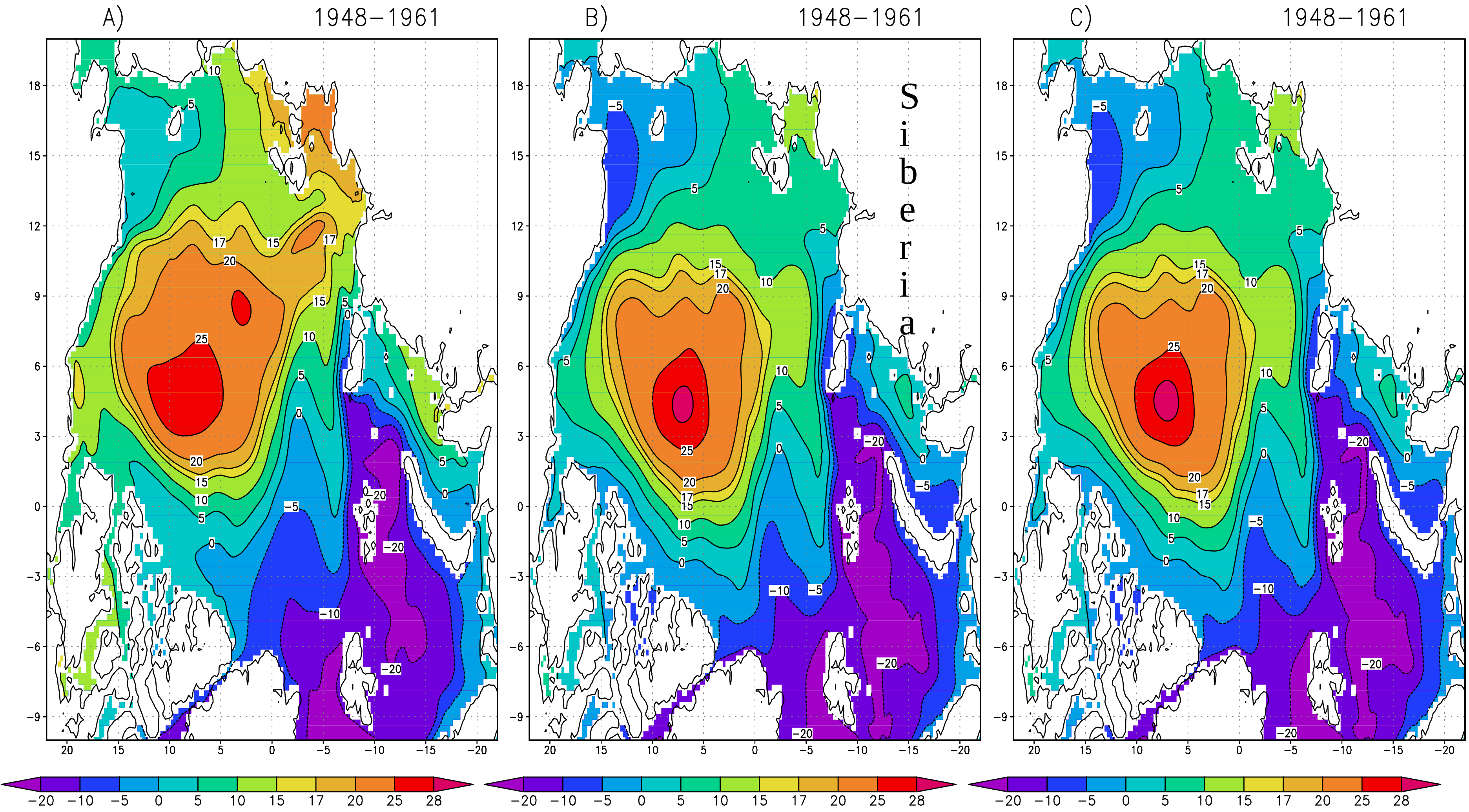
(2)



Sensitivity to mixing parameterizations.

Arctic Ocean SSH (cm plus 30). Model coordinate.

A) Pacanowski-Philander; B) Numerical solution k-omega; C) Asymptote the analytical solution

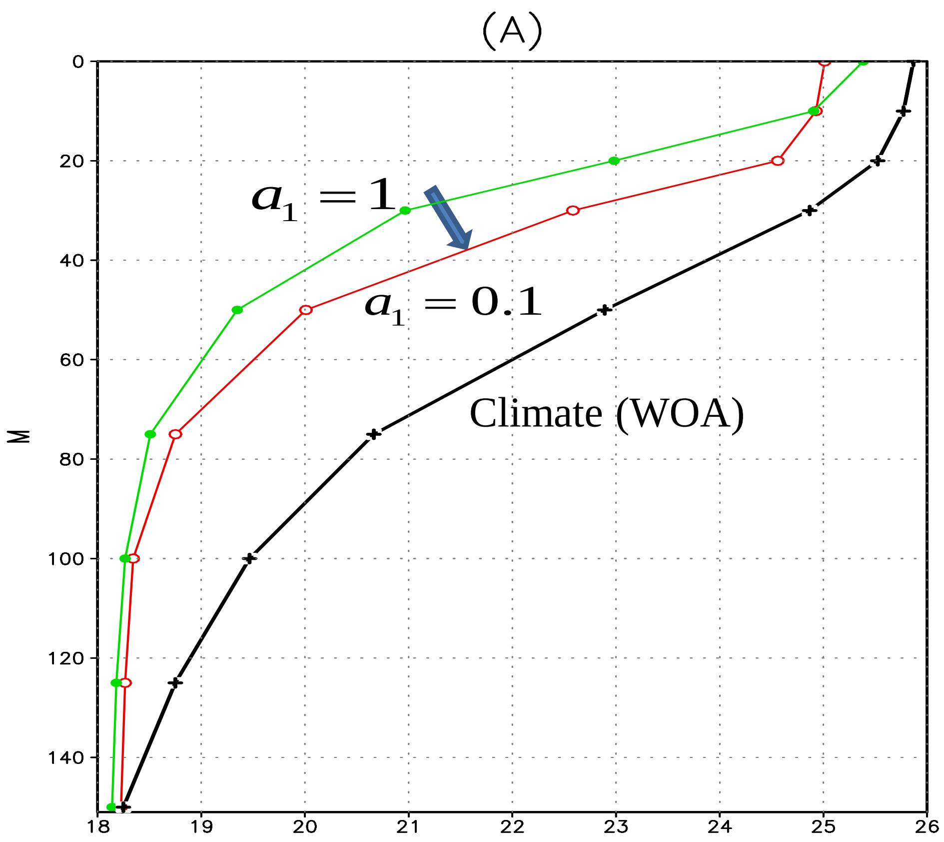


Possible perspectives.
Assimilation of climatic data in the equations for turbulence characteristics

$$A = (c_S^0)^{-1} \cdot \left(C_S^U \cdot G^2 - C_S^\rho \cdot N^2 \right), \qquad B = (c_S^0)^{-1} \cdot \left(c_1^\omega \cdot C_S^U \cdot G^2 - c_3^\omega \cdot C_S^\rho \cdot N^2 \right),$$

$$N^2 = \frac{1}{H} \frac{g}{\rho_0} \frac{\partial \rho_{pot}}{\partial \sigma}, \qquad \frac{\partial \rho_{pot}}{\partial \sigma} = a_1 \frac{\partial \rho_t}{\partial \sigma} + a_2 \frac{\partial \rho_{Cl}}{\partial \sigma}, \qquad a_2 = 1 - a_1,$$

ρ_t, ρ_{Cl} – Model and Mean Year Climate (WOA) potential water densities



Tropics. Local region:

August monthly mean potential temperature for 10 last years from 20 years experiment:
Green – no assimilation,
Red – variant with assimilation.

Black – August climate (WOA)

Conclusions

1. Splitting algorithm of the complete evolution equations for turbulence kinetic energy and the frequency of its dissipation by viscosity has been developed and used for a parameterization of the viscosity and diffusivity coefficients in the eddy-permitting ocean circulation model.

2. Equations of the turbulence model split into equations involving the transport–diffusion and generation–dissipation stage. A numerical explicit–implicit scheme of the solution, the analytical solution, and the asymptote of the analytical solution have been obtained at the generation–dissipation splitting stage.
3. Suggested splitting-equations model of the turbulence characteristics is as competitive in physical formulation as advanced differential models of developed turbulence, but its algorithm being much more efficient.

4. Parameterizations using the split model of turbulence provided a more adequate simulation of the temperature and salinity fields on decadal scales than the traditional parameterization of the viscosity and diffusivity coefficients, based on Richardson number. The computation time of the splitting-equations model compared to that of the simple parameterization incorporated in the circulation model. The parameterizations using the analytical solution and the numerical scheme at the step of the turbulence model for a generation–dissipation stage showed better quality in the simulation of the ocean climate than the most efficient parameterization using the asymptote of the analytical solution.
5. Eddy-permitting circulation model is very sensitive to the mixing parameterization. The high sensitivity is related to significant changes in the density fields in the upper part of the baroclinic oceanic layer.