Moving horizon estimation based model predictive control (MHE-MPC) for offset-free control of a single canal pool

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Introduction

- Model predictive control (MPC) is a powerful control option which is increasingly used by operational water managers for managing water systems.
- Closed loop performance of MPC is directly related to model accuracy. The **mismatch** between **real system** and **model** from **unknown disturbances** will result in **steady state offset** in the controlled output.
- <u>*Unknown Disturbances*</u>
  - Unknown water offtakes
  - Water loss (seepage, leakage, evaporation)
- Moving horizon estimation (MHE) is an optimization-based state estimation method working on an estimation window covering a certain number of past measurements.
- Moving horizon estimation based model predictive control (MHE-MPC) uses the past predictions of the model and the past measurements of the system to estimate unknown disturbances.

Test Canal – Internal Model

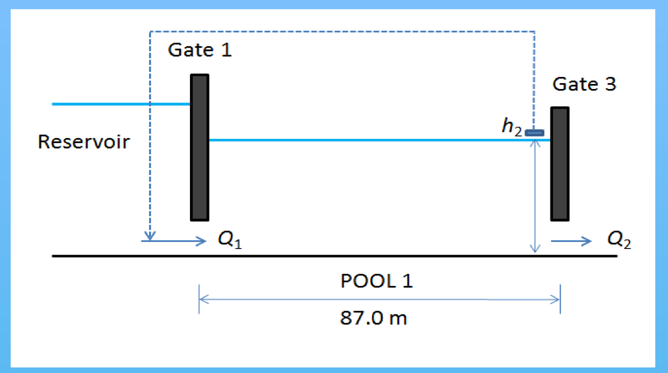


Fig.1 Schematization of a single canal pool with distant downstream controller

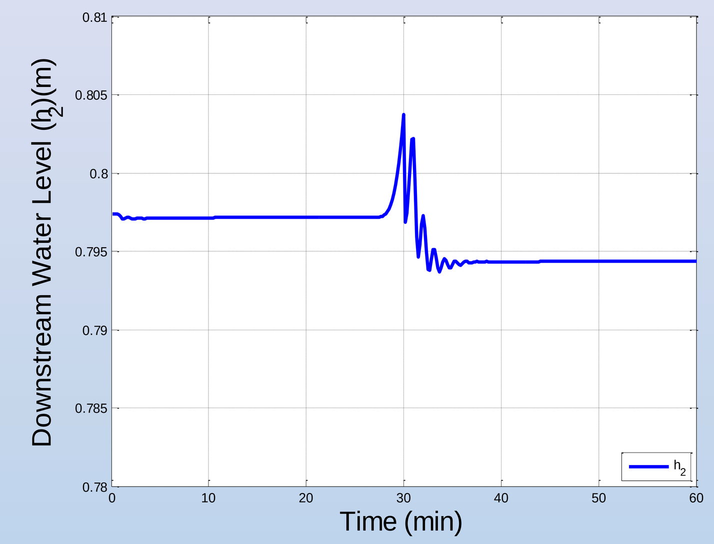
Table.1 Test canal parameters

Q_1 (m³/s)	0.010
As (m²)	38.28
ω_0 (rad/s)	0.101
M (-)	35.09

Integrator Resonance Model [2]

$$h_2(s)=\frac{\omega_0^2}{A_s s^3+\frac{s^2}{M}+A_s\cdot\omega_0^2\cdot s}\cdot Q_1(s)-\frac{2\cdot s^2+\frac{2}{A_s\cdot M}\cdot s+\omega_0^2}{A_s s^3+\frac{s^2}{M}+A_s\cdot\omega_0^2\cdot s}\cdot Q_2(s)$$

Offset Due to Unknown Disturbances



Set-point is 0.8 m, however due to unknown disturbances in the system, MPC can not achieve offset-free control.
The water level reached a steady state but with an undesired offset.

Fig. 2 Downstream water level (h_2) using MPC without offset free method

Moving Horizon Estimation and MHE-MPC

The internal model and the available output measurements over a given estimation window are used by MHE, for estimating the unknown disturbances (Q_{ext}) at the current time. The estimation is obtained by solving a least-squares problem over the estimation window, *N*, with an objective function that minimize the differences between the predicted water levels, $h_p(k)$ and the measured water levels $h_m(k)$:

$$\min_{Q_{ext}}\sum_{i=1}^{N}\left(h_m(i)-h_p(i)\right)^2$$

Where

$$h_p(s)=\frac{\omega_0^2}{A_s s^3+\frac{s^2}{M}+A_s\cdot\omega_0^2\cdot s}\cdot Q_1(s)-\frac{2\cdot s^2+\frac{2}{A_s\cdot M}\cdot s+\omega_0^2}{A_s s^3+\frac{s^2}{M}+A_s\cdot\omega_0^2\cdot s}\cdot (Q_2(s)+Q_{ext}(s))$$

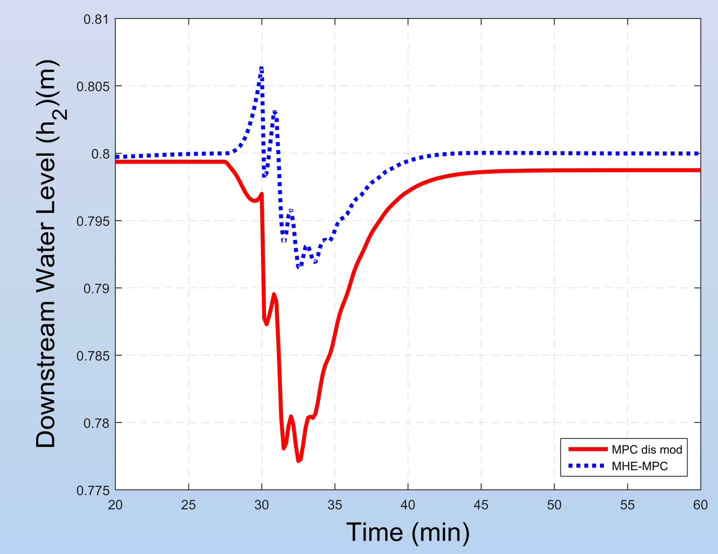
Estimation of Q_{ext} is added to the known disturbances in the MPC optimization

For comparison, an alternative offset-free scheme (MPC with disturbance modelling) is used. Disturbances were modelled by a disturbance model which augments the system states with integrating disturbances [3].

Simulation Results

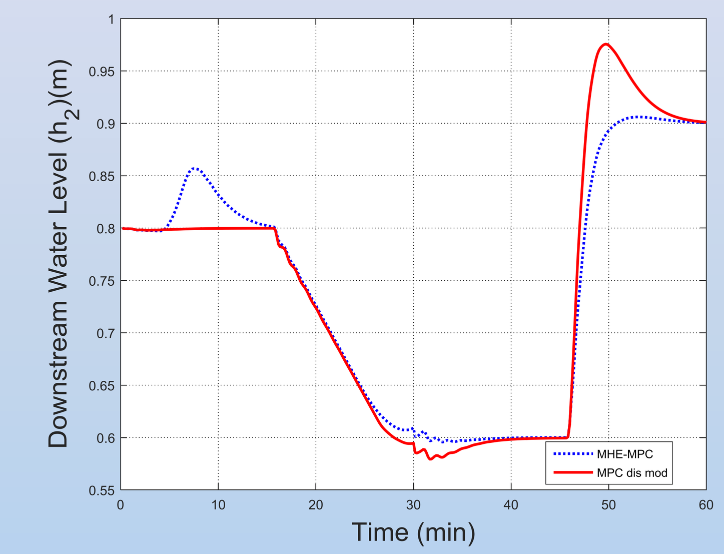
- 30 % Unknown Disturbance in Q_2

(a)



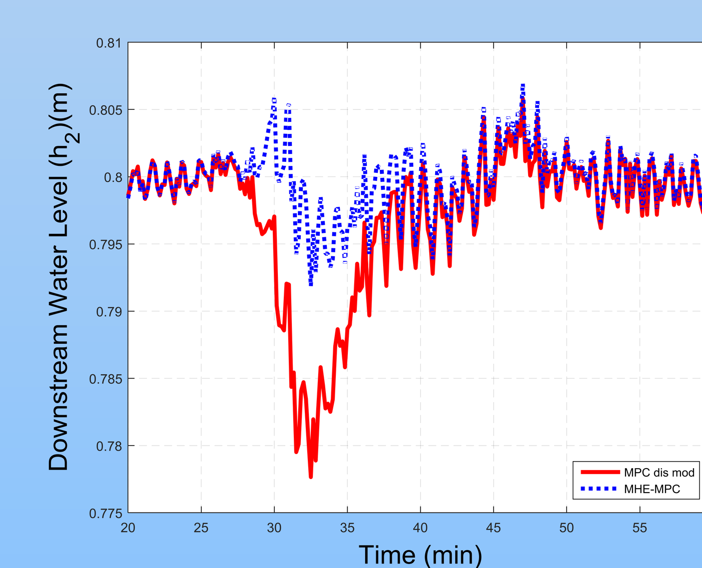
-Set-point change & 10 % unknown disturbance in Q_2

(b)



- Uncertain Unknown Disturbance in Q_2

(c)



-Set-point change & 200 % unknown disturbance in Q_2

(d)

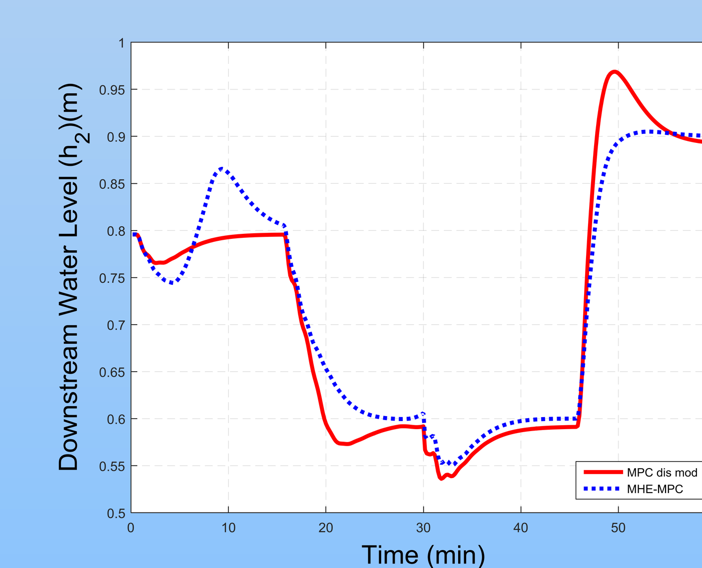


Fig. 3 Downstream water level (h_2) for 30% unknown disturbance in Q_2 (a); set point change & 10 % unknown disturbance in Q_2 (b); uncertain unknown disturbance in Q_2 (c); set point change & 200 % unknown disturbance in Q_2 (d)

Conclusion

- MHE-MPC estimates the unknown disturbance(s) with MHE and it is used to update the known disturbance for the MPC and the offset is removed.
- Simulation results show that the MHE-MPC is achieving better offset-free performance than the MPC with disturbance modeling scheme.
- MHE-MPC needs a certain amount of simulation time steps, length of the estimation window, to start unknown disturbance estimation. Therefore, the first step (*N*+1), MHE-MPC starts estimating the unknown disturbance, steep water level increases observed due to overshoots of the controlled inflow (Fig.3-b, Fig.3-d). Once the overshoots of inflow are resolved, MHE-MPC responds better to the known disturbances than the MPC with disturbance modeling.
- MHE-MPC achieves offset-free control in case of uncertain unknown disturbances (Fig.3-c).

References
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[2] van Overloop, P. J., Horváth, K., and Aydin, B. E. (2014). 'Model predictive control based on an integrator resonance model applied to an open water channel.' *Control Engineering Practice*, Elsevier, 27, 54–60.
[3] Aydin, B.E., van Overloop, P.J., Tian, X. (2014). 'Offset-free Model Predictive Control of an Open Water Reach.' *International Conference on Hydroinformatics*, New York.

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