



Fast Harmonic Splines and Parameter Choice Methods

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Solutions to boundary value problems in geoscience where the boundary is the Earth's surface are constructed in terms of harmonic splines. These are localizing trial functions that allow regional modeling or the improvement of a global model in a part of the Earth's surface.

Some cases of the occurring kernels can be equipped with a fast matrix-vector multiplication using the fast multipole method (FMM). The main idea of the fast multipole algorithm consists of a hierarchical decomposition of the computational domain into cubes and a kernel approximation for the more distant points. The numerical effort of the matrix-vector multiplication becomes linear in reference to the number of points for a prescribed accuracy of the kernel approximation.

This fast spline approximation which also allows the treatment of noisy data requires the choice of a smoothing parameter. We investigate several methods to (ideally automatically) choose this parameter with and without prior knowledge of the noise level. However, in order to keep a fast solution algorithm we do no longer have access to the whole matrix or e.g. its singular values whose computation requires a much larger numerical effort. This must be reflected by the parameter choice methods. Therefore, in some cases a further approximation is necessary.

The performance of these methods is considered for different types of noise in a large simulation study with applications to gravitational field modeling as well as to boundary value problems.