Geophysical Research Abstracts
Vol. 19, EGU2017-11635, 2017
EGU General Assembly 2017


# Early-time solution of the horizontal unconfined aquifer in the build-up phase

Elias Gravanis and Evangelos Akylas

Cyprus University of Technology, Civil Engineering and Geomatics, P.O. Box 50329, 3603, Limassol, Cyprus
(evangelos.akylas@cut.ac.cy, elias.gravanis@cut.ac.cy)

The Boussinesq equation is a dynamical equation for the free surface of saturated subsurface flows over an impervious bed. Boussinesq equation is non-linear. The non-linearity comes from the reduction of the dimensionality of the problem: The flow is assumed to be vertically homogeneous, therefore the flow rate through a cross section of the flow is proportional to the free surface height times the hydraulic gradient, which is assumed to be equal to the slope of the free surface (Dupuit approximation). In general, 'vertically' means normally on the bed; combining the Dupuit approximation with the continuity equation leads to the Boussinesq equation. There are very few transient exact solutions. Self- similar solutions have been constructed in the past by various authors. A power series type of solution was derived for a self-similar Boussinesq equation by Barenblatt in 1990. That type of solution has generated a certain amount of literature. For the unconfined flow case for zero recharge rate Boussinesq derived for the horizontal aquifer an exact solution assuming separation of variables. This is actually an exact asymptotic solution of the horizontal aquifer recession phase for late times. The kinematic wave is an interesting solution obtained by dropping the non-linear term in the Boussinesq equation. Although it is an approximate solution, and holds well only for small values of the Henderson and Wooding $\lambda$ parameter (that is, for steep slopes, high conductivity or small recharge rate), it becomes less and less approximate for smaller values of the parameter, that is, it is asymptotically exact with respect to that parameter.

In the present work we consider the case of the unconfined subsurface flow over horizontal bed in the build-up phase under constant recharge rate. This is a case with an infinite Henderson and Wooding parameter, that is, it is the limiting case where the non-linear term is present in the Boussinesq while the linear spatial derivative term goes away. Nonetheless, no analogue of the kinematic wave or the Boussinesq separable solution exists in this case. The late time state of the build-up phase under constant recharge rate is very simply the steady state solution. Our aim is to construct the early time asymptotic solution of this problem. The solution is expressed as a power series of a suitable similarity variable, which is constructed so that to satisfy the boundary conditions at both ends of the aquifer, that is, it is a polynomial approximation of the exact solution. The series turn out to be asymptotic and it is regularized by re-summation techniques which are used to define divergent series. The outflow rate in this regime is linear in time, and the (dimensionless) coefficient is calculated to eight significant figures. The local error of the series is quantified by its deviation from satisfying the self-similar Boussinesq equation at every point. The local error turns out to be everywhere positive, hence, so is the integrated error, which in turn quantifies the degree of convergence of the series to the exact solution.