

Non-Richardson Turbulent Particle Pair Diffusion

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Richardson's 1926 theory of turbulent particle pair diffusion [1] is based upon observational data from geophysical contexts and the hypothesis of locality, which leads to the turbulent pair diffusivity scaling, $K \sim \sigma_l^{4/3}$, where $\sigma_l(t) = \langle l(t)^2 \rangle^{1/2}$, and $l(t) = |x_1(t) - x_2(t)|$ is the distance between particle pair locations in an ensemble of particle pairs released at time $t = 0$ such that $l(0) = l_0 < \eta$. But Richardson's locality scaling has never been proven, as observed by Salazar Collins [2], "... there has not been an experiment that has unequivocally confirmed R-O scaling over a broad-enough range of time and with sufficient accuracy". Furthermore, a reappraisal of the 1926 dataset reveals that one of the data-points is from a molecular diffusion context; the remaining data from geophysical turbulence display an unequivocal non-local scaling, $K \sim \sigma_l^{1.564}$ [3] – a locality scaling of $4/3$ is not the best fit to the data.

Here, a new non-local theory based upon the principle that both local and non-local diffusional processes govern pair diffusion in homogeneous turbulence has been developed. Using a novel mathematical approach based upon the Fourier decomposition of the pair relative velocity, the theory is developed in the context of generalized power law energy spectra over extended inertial subranges, $E(k) \sim k^{-p}$ for $1 < p \leq 3$. The theory predicts the scaling, $K \sim \sigma_l^{\gamma_p}$, with γ_p intermediate between the purely local and the purely non-local scalings, i.e. $(1+p)/2 < \gamma_p \leq 2$. A Lagrangian diffusion model, Kinematic Simulations [4,5] is used to examine the predictions of the new non-local theory. For the pair diffusivity KS produces the scalings, $K \sim \sigma_l^{1.545}$ to $K \sim \sigma_l^{1.57}$, in the accepted range of realistic intermittent turbulence spectra, $E(k) \sim k^{-1.72}$ to $k^{-1.74}$ with inertial subrange $1 \leq k \leq 10^6$ – the result for $E \sim k^{1.74}$ in particular is in remarkably close agreement with the revised 1926 dataset mentioned above; note that $K \sim \sigma_l^{1.57}$ is equivalent to $\langle l(t)^2 \rangle \sim t^{4.65}$.

The non-local theory provides a new picture for the turbulent particle pair diffusion process. The consequences of non-locality for the general theory of turbulence is the subject of active investigation by the author.

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References

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