

Turbulent Inertial Particle Pair Diffusion

Syed Usama and Nadeem Malik

King Fahd University of Petroleum & Minerals, Department of Mathematics & Statistics, P.O. Box 5046, 31261 Dhahran, Saudi Arabia (namalik@kfupm.edu.sa)

Inertial particle pair diffusion has received much less attention than fluid particle pair diffusion, even though it is arguably more relevant to real world applications, such as sand storms, and pollen dispersion. Only the DNS work of Bec et al [1] has been reported. A non-local theory of fluid particle pair diffusion has recently been proposed [2,3]; but the question is, can non-locality be extended to inertial particle pair diffusion? Here, we investigate it using Kinematic Simulations [4,5], in the limit of Stokes' drag where the transport is given by,

$$\frac{dx}{dt} = v(t), \quad \frac{dv}{dt} = -\frac{1}{\tau}(v(t) - u) \quad (1)$$

$x(t)$ is the particle position at time t , $v(t)$ is the particle velocity, $u(x, t)$ is the Eulerian velocity field generated by the KS model, τ is the particle response time. The Stokes number is, $St = \tau/t_\eta$, where t_η is the Kolmogorov time scale, $\sigma_l(t) = \langle l(t)^2 \rangle^{1/2}$, where $l(t) = |x_1(t) - x_2(t)|$ is the distance between particles in a pair, in an ensemble of particle pairs released at time $t = 0$ such that $l(t = 0) = l_0 < \eta$. η is the Kolmogorov scale of the turbulence.

For short times, the energy in the small scales of turbulence does not affect the particle relative motion, thus we expect ballistic motion, $K_I \sim \sigma_l^1$, for $t \ll T^*$, where T^* is some time scale. At large pair separations the turbulent energy will be dominant, and the diffusion will approach the non-local fluid pair diffusion law, $K_I \rightarrow K \sim \sigma_l^{1.53}$, for $T^* \ll t \ll T_L$, [2,3] where T_L is a timescale when their motions are independent. We expect an intermediate transition regime determined by when the local Stokes number, $St(l^*) = \tau/T^* = 1$. According to the locality theory of Bec et al [1], this leads to, $\sigma_l^*/\eta \sim St^{2/3}$. However, according to the non-local theory [2,3], the scaling should be higher, $\sigma_l^*/\eta \sim St^\beta$ with $\beta > 2/3$.

KS was used in a frame of reference moving with the (virtual) large scale sweeping velocities with spectrum, $E(k) \sim k^{-5/3}$, for $1 \leq k \leq 10^4$, and $E(k) = 0$, for $k < 1$, and for a wide range of Stokes number, $0 \leq St \leq 50$. The results display short time ballistic regimes, and long-time non-local scaling, $K_I \rightarrow K \sim \sigma_l^{1.53}$, and an intermediate transition regime. Furthermore, σ_l^*/η displays a scaling which is approximately $\sigma_l^*/\eta \sim St^{0.75}$. All of these results are in line with the predictions of the non-local theory for inertial particle pair diffusion.

This work is important because it addresses the fundamentals of inertial pair diffusion, and because of its potential to yield modeling strategies for applications in practical contexts noted above. This work will be completed for different spectra, $E(k) \sim k^{-p}$, for $1 < p \leq 3$, and for different sizes of the inertial subrange.

References

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