



## **Four-dimensional ensemble variational data assimilation and the unstable subspace**

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The performance of (ensemble) Kalman filters used for data assimilation in the geosciences critically depends on the dynamical properties of the evolution model. A key aspect, emphasized in the seminal work of Anna Trevisan and co-authors, is that the error covariance matrix is asymptotically supported by the unstable-neutral subspace only, i.e. it is spanned by the backward Lyapunov vectors with non-negative exponents. The analytic proof of such a property for the Kalman filter error covariance has been recently given, and in particular that of its confinement to the unstable-neutral subspace.

In this paper, we first generalize those results to the case of the Kalman smoother in a linear, Gaussian and perfect model scenario. We also provide square-root formulae for the filter and smoother that make the connection with ensemble formulations of the Kalman filter and smoother, where the span of the error covariance is described in terms of the ensemble deviations from the mean.

We then discuss how this neat picture is modified when the dynamics are nonlinear and chaotic, and for which analytic results are precluded or difficult to obtain. A numerical investigation is carried out to study the approximate confinement of the anomalies for both a deterministic ensemble Kalman filter (EnKF) and a four-dimensional ensemble variational method, the iterative ensemble Kalman smoother (IEnKS), in a perfect model scenario. The confinement is characterized using geometrical angles that determine the relative position of the anomalies with respect to the unstable-neutral subspace. The alignment of the anomalies and of the unstable-neutral subspace is more pronounced when observation precision or frequency, as well as the data assimilation window length for the IEnKS, are increased. This leads to the increase of the data assimilation accuracy and shows that, under perfect model assumptions, spanning the full unstable-neutral subspace is sufficient to achieve satisfactory performance. These results also suggest that the IEnKS and the deterministic EnKF realize in practice (albeit implicitly) the paradigm behind the approach of Anna Trevisan and co-authors known as the assimilation in the unstable subspace.