

What does the structure of its visibility graph tell us about the nature of the time series?

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Visibility graphs are a recently introduced method to construct complex network representations based upon univariate time series in order to study their dynamical characteristics [1]. In the last years, this approach has been successfully applied to studying a considerable variety of geoscientific research questions and data sets, including non-trivial temporal patterns in complex earthquake catalogs [2] or time-reversibility in climate time series [3]. It has been shown that several characteristic features of the thus constructed networks differ between stochastic and deterministic (possibly chaotic) processes, which is, however, relatively hard to exploit in the case of real-world applications.

In this study, we propose studying two new measures related with the network complexity of visibility graphs constructed from time series, one being a special type of network entropy [4] and the other a recently introduced measure of the heterogeneity of the network's degree distribution [5]. For paradigmatic model systems exhibiting bifurcation sequences between regular and chaotic dynamics, both properties clearly trace the transitions between both types of regimes and exhibit marked quantitative differences for regular and chaotic dynamics. Moreover, for dynamical systems with a small amount of additive noise, the considered properties demonstrate gradual changes prior to the bifurcation point. This finding appears closely related to the subsequent loss of stability of the current state known to lead to a critical slowing down as the transition point is approached. In this spirit, both considered visibility graph characteristics provide alternative tracers of dynamical early warning signals consistent with classical indicators.

Our results demonstrate that measures of visibility graph complexity (i) provide a potentially useful means to tracing changes in the dynamical patterns encoded in a univariate time series that originate from increasing autocorrelation and (ii) allow to systematically distinguish regular from deterministic-chaotic dynamics. We demonstrate the application of our method for different model systems as well as selected paleoclimate time series from the North Atlantic region. Notably, visibility graph based methods are particularly suited for studying the latter type of geoscientific data, since they do not impose intrinsic restrictions or assumptions on the nature of the time series under investigation in terms of noise process, linearity and sampling homogeneity.

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