



Uncertainty Quantification for Adjoint-Based Inverse Problems with Sparse Data

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The adjoint method of data assimilation (DA) is used in many fields of Geosciences. It fits a dynamical model to observations in a least-squares optimization problem, leading to a solution that follows the model equations exactly. While the physical consistency of the obtained solution makes the adjoint method an attractive DA technique for many applications, one of its major drawbacks is that an accompanying uncertainty quantification is computationally challenging.

In theory, the Hessian of the model-data misfit function can provide such an error estimate on the solution of the inverse problem because - under certain assumptions - it can be associated with the inverse of the error covariance matrix. In practice, however, studies that use adjoint-based DA into ocean GCMs usually don't deal with a quantification of uncertainties, mostly because an analysis of the Hessian is often intractable due to its high dimensionality.

This work is motivated by the fact that an increasing number of studies apply the adjoint-based DA machinery to paleoceanographic problems - without considering accompanying uncertainties. In such applications, the number of observations can be of the order 10^2 , while the dimension of the control space is still as high as of the order 10^6 to 10^8 . An uncertainty quantification in such heavily underdetermined inverse problems seems even more crucial, an objective that we pursue here. We take advantage of the fact that in such situations the Hessian is of very low rank (while still of high dimension). This enables us to explore in great detail to what extent paleo proxy data from ocean sediment cores informs the solution of the inverse problem.

We use the MIT general circulation model (MITgcm) and sample a sparse set of observations from a control simulation, corresponding to available data from ocean sediment cores. We then quantify how well the synthetic data constrains different quantities of interest, such as heat content of specific ocean basins or volume/heat transport at various latitudes. Furthermore, we study how prior information and data uncertainties influence the error quantification.