

Overland flow connectivity on planar patchy hillslopes – modified percolation theory approaches and combinatorial model of urns

David Nezlobin (1), Sarah Pariente (2), Hanoch Lavee (2), and Eyal Sachs (2)

(1) Faculty of Civil and Environmental Engineering, Technion, Haifa, Israel, (2) Geography and Environment Department, Bar Ilan University, Israel

Source-sink systems are very common in hydrology; in particular, some land cover types often generate runoff (e.g. embedded rocks, bare soil), while others obstruct it (e.g. vegetation, cracked soil). Surface runoff coefficients of patchy slopes/plots covered by runoff generating and obstructing covers (e.g., bare soil and vegetation) depend critically on the percentage cover (i.e. sources/sinks abundance) and decrease strongly with observation scale. The classic mathematical percolation theory provides a powerful apparatus for describing the runoff connectivity on patchy hillslopes, but it ignores the strong effect of the overland flow directionality. To overcome this and other difficulties, modified percolation theory approaches can be considered, such as straight percolation (for the planar slopes), quasi-straight percolation and models with limited obstruction. These approaches may explain both the observed critical dependence of runoff coefficients on percentage cover and their scale decrease in systems with strong flow directionality (e.g. planar slopes). The contributing area increases sharply when the runoff generating percentage cover approaches the straight percolation threshold. This explains the strong increase of the surface runoff and erosion for relatively low values (normally less than 35%) of the obstructing cover (e.g., vegetation).

Combinatorial models of urns with restricted occupancy can be applied for the analytic evaluation of meaningful straight percolation quantities, such as NOGA's (Non-Obstructed Generating Area) expected value and straight percolation probability. It is shown that the nature of the cover-related runoff scale decrease is combinatorial – the probability for the generated runoff to avoid obstruction in unit area decreases with scale for the non-trivial percentage cover values. The magnitude of the scale effect is found to be a skewed non-monotonous function of the percentage cover. It is shown that the cover-related scale effect becomes less prominent if the obstructing capacity decreases, as generally occurs during heavy rainfalls.

The plot width has a moderate positive statistical effect on runoff and erosion coefficients, since wider patchy plots have, on average, a greater normalized contributing area and a higher probability to have runoff of a certain length. The effect of plot width depends by itself on the percentage cover, plot length, and compared width scales. The contributing area uncertainty brought about by cover spatial arrangement is examined, including its dependence on the percentage cover and scale.

In general, modified percolation theory approaches and combinatorial models of urns with restricted occupancy may link between the critical dependence of runoff on percentage cover, cover-related scale effect, and statistical uncertainty of the observed quantities.