



## **Nonlinear and subharmonic stability analysis in film-driven geomorphological patterns**

Matteo Bertagni and Carlo Camporeale

Politecnico di Torino, Torino, Italy (matteo.bertagni@polito.it)

Similar fascinating wavy patterns can be found widespread both in caves and ice-falls, as a result of the interaction of a gravity-driven water film with a deformable surface (calcite or ice). Thanks to such a remarkable parallelism, a unified approach is adopted in the study of patterns formation of longitudinally oriented organ-pipe-like structures, called flutings. However, the morphological time scales differentiate noticeably the two cases: in fact, while in the ice-falls, flutings fully develop within some days, cave patterns can evolve for millenia, gaining an additional value as the silent repository of the past climate. In the present work, flutings formation has been studied both linearly and nonlinearly with the aid of two novel techniques, in the context of morphodynamics: gradient expansion and center manifold projection. In this way, closed form relationships for the selected wavenumber and for the finite amplitude have been achieved, thus defining the flutings spatial structure in the parameters space in a complete way. However, finite amplitude monochromatic waves may be destabilised by nonlinear interactions with other modes. In particular, in the present case, the range of unstable wavenumbers spans from zero to a finite value; hence the  $1/2$  subharmonic of any excited wave within this range is linearly unstable, including the  $1/2$  subharmonic of the fundamental. We find that this condition, despite different statements in previous works, is not sufficient to trigger the instability and that the fundamental mode results stable to subharmonic disturbances.