



On existence and uniqueness of solutions for variational data assimilation

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Data assimilation is a term from the geosciences and refers to methods for estimating orbits of dynamical models from observations. Variational techniques for data assimilation estimate these orbits by minimising an appropriate cost functional which takes the error with respect to the observations but also deviations of the orbits from the model equations into account. Such techniques are very important in practice. In this contribution, the problem of existence and uniqueness of solutions to variational data assimilation is investigated. Under mild hypotheses a solution to this problem exists. The problem of uniqueness is investigated as well, and several results (which all have analogues in optimal control) are established in the present context. The value function is introduced as the cost of an optimal trajectory starting from a given initial condition. The necessary conditions in combination with an envelope theorem can be used to demonstrate that the solution is unique if and only if the value function is differentiable at the given initial condition. This occurs for all initial conditions except maybe on a set of Lebesgue measure zero. Several examples are studied which demonstrate that non-uniqueness of solutions cannot be ruled out altogether though, which has important consequences in practice.

References: J. Bröcker, "Existence and Uniqueness For Four Dimensional Variational Data Assimilation in Discrete Time.", *SIAM Journal of Applied Dynamical Systems* (accepted).