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# Weak solution concept and Galerkin's matrix for the exterior of an oblate ellipsoid of revolution in the representation of the Earth's gravity potential by buried masses

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The paper is motivated by the role of boundary value problems in Earth's gravity field studies. The discussion focuses on Neumann's problem formulated for the exterior of an oblate ellipsoid of revolution as this is considered a basis for an iteration solution of the linear gravimetric boundary value problem in the determination of the disturbing potential. The approach follows the concept of the weak solution and Galerkin's approximations are applied. This means that the solution of the problem is approximated by linear combinations of basis functions with scalar coefficients. The construction of Galerkin's matrix for basis functions generated by elementary potentials (point masses) is discussed. Ellipsoidal harmonics are used as a natural tool and the elementary potentials are expressed by means of series of ellipsoidal harmonics. The problem, however, is the summation of the series that represent the entries of Galerkin's matrix. It is difficult to reduce the number of summation indices since in the ellipsoidal case there is no analogue to the addition theorem known for spherical harmonics. Therefore, the straightforward application of series of ellipsoidal harmonics is complemented by deeper relations contained in the theory of ordinary differential equations of second order and in the theory of Legendre's functions. Subsequently, also hypergeometric functions and series are used. Moreover, within some approximations the entries are split into parts. Some of the resulting series may be summed relatively easily, apart from technical tricks. For the remaining series the summation was converted to elliptic integrals. The approach made it possible to deduce a closed (though approximate) form representation of the entries in Galerkin's matrix. The result rests on concepts and methods of mathematical analysis. In the paper it is confronted with a direct numerical approach applied for the implementation of Legendre's functions. The computation of the entries is more demanding in this case, but conceptually it avoids approximations. Finally, some specific features associated with function bases generated by elementary potentials in case the ellipsoidal solution domain are illustrated and discussed.