

Study of dispersive and nonlinear effects of coastal wave dynamics with a fully nonlinear potential flow model

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Efficient and accurate numerical models simulating wave propagation are required for a variety of engineering projects including the evaluation of coastal risks, the design of protective coastal structures, and the estimation of the potential for marine renewable energy devices. Nonlinear and dispersive effects are particularly significant in the coastal zone where waves interact with the bottom, the shoreline, and coastal structures. The main challenge in developing a numerical models is finding a compromise between computational efficiency and the required accuracy of the simulated wave field.

Here, a potential approach is selected and the (fully nonlinear) water wave problem is formulated using the Euler-Zakharov equations (Zakharov, 1968) describing the temporal evolution of the free surface elevation and velocity potential. The proposed model (Yates and Benoit, 2015) uses a spectral approach in the vertical (i.e. the vertical variation of the potential is approximated by a linear combination of the first $NT+1$ Chebyshev polynomials, following the work of Tian and Sato (2008)). The Zakharov equations are integrated in time using a fourth-order Runge-Kutta scheme with a constant time step. At each sub-timestep, the Laplace Boundary Value Problem (BVP) is solved to estimate the free surface vertical velocity using the spectral approach, with typical values of NT between 5 to 8 for practical applications.

The 1DH version of the code is validated with comparisons to the experimental data set of Becq-Girard et al. (1999), which studied the propagation of irregular waves over a beach profile with a submerged bar. The non-linear and dispersive capacities of the model are verified with the correct representation of wave-wave interactions, in particular the transfer of energy between different harmonic components during wave propagation (analysis of the transformation of the variance spectrum along the channel). Evolution of wave skewness, asymmetry and kurtosis along the bathymetric profile also compare well with the measured values. The statistical distributions of the free surface elevation and wave height, calculated from the simulated time series, are compared to those of the measurements, with particular attention paid to the extreme waves.

To use this model for realistic cases with complex bathymetric variations and multidirectional wave fields, the model has been extended to two horizontal dimensions (2DH). The spectral approach in the vertical dimension is retained, while the horizontal plane is discretized with scattered nodes to maintain the model's flexibility. The horizontal derivatives are estimated with finite-difference type formulas using Radial Basis Functions (Wright and Fornberg, 2006). The 2DH version of the code is applied to simulate the propagation of regular waves over a semi-circular step, which acts as a focusing lens. The simulation results are compared to the experimental data set of Whalin (1971). The evolution of the higher harmonic amplitudes in the shallow-water zone demonstrates the ability of the model to simulate wave propagation over complex 2DH coastal bathymetries.

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