Geophysical Research Abstracts
Vol. 19, EGU2017-16558, 2017
EGU General Assembly 2017

The limiting form of symmetric instability in geophysical flows

Stephen Griffiths
Department of Applied Mathematics, University of Leeds, Leeds, United Kingdom (s.d.griffiths@leeds.ac.uk)

The stability of parallel flow with vertical shear, density stratification and background rotation is of fundamental importance in geophysical fluid dynamics. For a flow with vertical shear U_z and buoyancy frequency N, the dominant instability is typically a symmetric instability (sometimes known as slantwise convection) when $1/4 < \mathrm{Ri} \lesssim 1$, where $\mathrm{Ri} = N^2/U_z^2$. Symmetric instability, which in its simplest form has no along-stream variations, is known to be active in both the troposphere and upper ocean.

The corresponding (symmetric) inviscid linear stability problem has been well studied for the case of constant U_z and N, and has some interesting mathematical properties (e.g., non-separable governing PDE, an absence of normal mode solutions in rectangular domains). Here, for the first time, a general theory of symmetric instability is given when Ri varies smoothly with height, thinking of the more realistic case where an unstable layer with $\mathrm{Ri} < 1$ lies between two stable layers with $\mathrm{Ri} > 1$. The mathematical theory is developed for horizontally periodic disturbances to a basic state with arbitrary smooth $N(z)$, but constant U_z. An asymptotic analysis is used to derive expressions for the most unstable mode, which occurs in the limit of large cross-isentropic wavenumber and takes the form of solutions trapped within the unstable layer; the same result is derived using an interesting generalised parcel dynamics argument, which explicitly shows how the trapping is linked to vertical variations of the potential vorticity. A separate asymptotic analysis is given for the small wavenumber limit, where only one such trapped mode may exist, as expected from the spectral theory of the Schrödinger equation. These two limiting results are shown to be consistent with an exact solution of the linear stability problem that can be obtained for a special choice of $N(z)$.

The asymptotic analysis can be extended to allow for weak diffusion at arbitrary Prandtl number, yielding an explicit diffusive scale selection at large wavenumber. Numerical simulations show that these weakly diffusive modes dominate the early stages of the nonlinear evolution of the symmetric instability.