



Approximate dispersion relation for surface waves on current with arbitrary depth dependence

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We present a new approximate dispersion relation for surface waves propagating at arbitrary direction with a horizontal shear current whose magnitude and direction may vary arbitrarily with depth, and compare it with existing approximations, in particular (the leading order correction of) the widely used approximation by Kirby & Chen [1989]. There is no practical difference in calculational complexity and effort between the new model and that of Kirby & Chen (KC).

We derive and analyse for the first time to our knowledge, the specific criteria that must be satisfied to ensure the applicability and accuracy of the KC formula, as well as for our new approximation. The analysis shows how conditions of applicability for the new approximation are less strict than for the KC approximation, making the new formula more widely applicable. The new approximation is always applicable whenever the KC formula is, in which case the two approximations coincide to leading order in a small parameter. Our analysis moreover explains why the KC approximation works well (apparently serendipitously) even in situations where the assumptions from which it was originally derived are strongly violated.

We argue that the new approximation is more widely applicable, and can be employed with greater confidence than the widely used KC approximation due to better robustness: it is accurate in several realistic example flows where the KC model fails, it remains reasonable in particularly difficult cases where the KC model gives nonsensical results, unlike the KC approximation it is exact for flows of linear depth dependence, and it provides the possibility of a higher order approximation which, unlike existing higher-order schemes, may be applied across the wave spectrum rather than restricted to particular asymptotic regimes.