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Wave-current interactions in three dimensions: why 3D radiation stresses are not practical

Fabrice Ardhuin
CNRS, Laboratoire d'Océanographie Physique Spatiale, Plouzané, France (ardhuin@ifremer.fr)

The coupling of ocean circulation and wave models is based on a wave-averaged mass and momentum conservation equations. Whereas several equivalent equations for the evolution of the current momentum have been proposed, implemented, and used, the possibility to formulate practical equations for the total momentum, which is the sum of the current and wave momenta, has been obscured by a series of publications. In a recent update on previous derivations, Mellor (J. Phys. Oceanogr. 2015) proposed a new set of wave-forced total momentum equations. Here we show that this derivation misses a term that integrates to zero over the vertical. This is because he went from his depth-integrated eq. (28) to the 3D equation (30) by simply removing the integral, but any extra zero-integrating term can be added.

Corrected for this omission, the equations of motion are equivalent to the earlier equations by Mellor (2003) which are correct when expressed in terms of wave-induced pressure, horizontal velocity and vertical displacement. Namely the total momentum evolution is driven by the horizontal divergence of a horizontal momentum flux,

$$S_{\alpha\beta} = \overline{\widetilde{u}_\alpha \widetilde{u}_\beta} + \delta_{\alpha\beta} \overline{\frac{\partial \widetilde{s}}{\partial \varsigma} (\widetilde{p} - g\widetilde{s})} \tag{1}$$

and the vertical divergence of a vertical flux,

$$S_{\alpha z} = \overline{(\widetilde{p} - g\widetilde{s}) \partial \widetilde{s} / \partial x_\alpha}, \tag{2}$$

where $\widetilde{p}$ is the wave-induced non-hydrostatic pressure, $\widetilde{s}$ is the wave-induced vertical displacement, and $\widetilde{u}_\alpha$ is the horizontal wave-induced velocity in direction α.

So far, so good. Problems arise when $\widetilde{p}$ and $\widetilde{s}$ are evaluated. Indeend, Ardhuin et al. (J. Phys. Oceanogr. 2008) showed that, over a sloping bottom $\partial S_{\alpha\beta}/\partial x_\beta$ is of order of the slope, hence a consistent wave forcing requires an estimation of $S_{\alpha z}$ that must be estimated to first order in the bottom slope. For this, Airy wave theory, i.e.

$$\widetilde{p} \simeq ga \frac{\cosh(kz + kh)}{\cosh(kD)} \cos\psi, \tag{3}$$

is not enough. Ardhuin et al. (2008) has shown that using an exact solution of the Laplace equations the vertical flux can indeed be computed. The alternative of neglecting completely $S_{\alpha z}$, as suggested by Mellor (2011) for small slopes, will always generate spurious currents because of the unbalanced forcing $\partial S_{\alpha\beta}/\partial x_\beta$.

Fortunately, there are many explicit versions of the wave-averaged equations without the wave momentum in them (Suzuki and Fox-Kemper 2016), with or without vortex force which are all consistent with the exact 3D equations of Andrews and McIntyre (1978). There is thus no need to stumble again and again on this fundamental problem of vertical momentum flux, which is a flux of wave momentum. The problem simply goes away by writing the equations for the current momentum only, without the problematic wave momentum. The current and wave momentum are coupled by forcing terms, and the wave momentum can be solved in 2D, the vertical distribution of momentum being maintained by the complex flux $S_{\alpha z}$.