Laplacian versus topography in the solution of the linear gravimetric boundary value problem by means of successive approximations

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The aim of this paper is to discuss the solution of the linearized gravimetric boundary value problem by means of the method of successive approximations. We start with the relation between the geometry of the solution domain and the structure of Laplace’s operator. Similarly as in other branches of engineering and mathematical physics a transformation of coordinates is used that offers a possibility to solve an alternative between the boundary complexity and the complexity of the coefficients of the partial differential equation governing the solution. Laplace’s operator has a relatively simple structure in terms of ellipsoidal coordinates which are frequently used in geodesy. However, the physical surface of the Earth substantially differs from an oblate ellipsoid of revolution, even if it is optimally fitted. Therefore, an alternative is discussed. A system of general curvilinear coordinates such that the physical surface of the Earth is embedded in the family of coordinate surfaces is used. Clearly, the structure of Laplace’s operator is more complicated in this case. It was deduced by means of tensor calculus and in a sense it represents the topography of the physical surface of the Earth. Nevertheless, the construction of the respective Green’s function is more simple, if the solution domain is transformed. This enables the use of the classical Green’s function method together with the method of successive approximations for the solution of the linear gravimetric boundary value problem expressed in terms of new coordinates. The structure of iteration steps is analyzed and where useful also modified by means of the integration by parts. Comparison with other methods is discussed.