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Initial-value problem for the Gardner equation applied to nonlinear internal waves

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The Gardner equation is a fundamental mathematical model for the description of weakly nonlinear weakly dispersive internal waves, when cubic nonlinearity cannot be neglected. Within this model coefficients of quadratic and cubic nonlinearity can both be positive as well as negative, depending on background conditions of the medium, where waves propagate (sea water density stratification, shear flow profile) [Rouvinskaya et al., 2014, Kurkina et al., 2011, 2015].

For the investigation of weakly dispersive behavior in the framework of nondimensional Gardner equation with fixed (positive) sign of quadratic nonlinearity and positive or negative cubic nonlinearity

$$\frac{\partial \eta}{\partial t} + 6\eta\,(1 \pm \eta)\,\frac{\partial \eta}{\partial x} + \frac{\partial^3 \eta}{\partial x^3} = 0, \tag{1}$$

the series of numerical experiments of initial-value problem was carried out for evolution of a bell-shaped impulse of negative polarity (opposite to the sign of quadratic nonlinear coefficient):

$$\eta(x, t = 0) = -a\mathrm{sech}^2\left(\frac{x}{x_0}\right), \tag{2}$$

for which amplitude a and width x_0 was varied. Similar initial-value problem was considered in the paper [Trillo et al., 2016] for the Korteweg – de Vries equation. For the Gardner equation with different signs of cubic nonlinearity the initial-value problem for piece-wise constant initial condition was considered in detail in [Grimshaw et al., 2002, 2010].

It is widely known, for example, [Pelinovsky et al., 2007], that the Gardner equation (1) with negative cubic nonlinearity has a family of classic solitary wave solutions with only positive polarity,and with limiting amplitude equal to 1. Therefore evolution of impulses (2) of negative polarity (whose amplitudes a were varied from 0.1 to 3, and widths at the level of $a/2$ were equal to triple width of solitons with the same amplitude for $a < 1$, and triple width of soliton with amplitude 0.9999 for $a > 1$) was going on a universal scenario with the generation of nonlinear Airy wave.

For the Gardner equation (1) with the positive cubic nonlinearity coefficient there exist two one-parametric families of solitons (family with positive polarity, and family with negative polarity bounded below by the amplitude of 2) and two-parametric family of breathers (oscillatory wave packets). In this case varying amplitude and width of bell-shaped initial impulse leads to plenty of different evolutionary scenarios with the generation of solitary waves, breathers, solibores and nonlinear Airy wave in their various combinations.

Statistical analysis of the wave field in time shows almost permanent substantial exceedance of the level of the significant wave height in some position in spatial coordinate. Evolution of Fourier spectrum of the wave field is also analyzed, and its behavior after a long time of initial wave evolution demonstrates the power asymptotic for small wave numbers and exponential asymptotic for large wave numbers.

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