

Pseudo-incompressible, finite-amplitude gravity waves: wave trains and stability

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Based on weak asymptotic WKB-like solutions for two-dimensional atmospheric gravity waves (GWs) traveling wave solutions (wave trains) are derived and analyzed with respect to stability.

A systematic multiple-scale analysis using the ratio of the dominant wavelength and the scale height as a scale separation parameter is applied on the fully compressible Euler equations. A distinguished limit favorable for GWs close to static instability, reveals that pseudo-incompressible rather than Boussinesq theory applies. A spectral expansion including a mean flow, combined with the additional WKB assumption of slowly varying phases and amplitudes, is used to find general weak asymptotic solutions. This ansatz allows for arbitrarily strong, non-uniform stratification and holds even for finite-amplitude waves. It is deduced that wave trains as leading order solutions can only exist if either some non-uniform background stratification is given but the wave train propagates only horizontally or if the wave train velocity vector is given but the background is isothermal. For the first case, general analytical solutions are obtained that may be used to model mountain lee waves. For the second case with the additional assumption of horizontal periodicity, upward propagating wave train fronts were found. These wave train fronts modify the mean flow beyond the non-acceleration theorem. Stability analysis reveal that they are intrinsically modulationally unstable. The range of validity for the scale separation parameter was tested with fully nonlinear simulations. Even for large values an excellent agreement with the theory was found.