

A fast Laplace solver approach to pore scale permeability

Christoph Arns (1) and Pierre Adler (2)

(1) Australia (c.arns@unsw.edu.au), (2) France (pierre.adler@upmc.fr)

The permeability of a porous medium can be derived by solving the Stokes equations in the pore space with no slip at the walls. The resulting velocity averaged over the pore volume yields the permeability K_S by application of the Darcy law.

The Stokes equations can be solved by a number of different techniques such as finite differences, finite volume, Lattice Boltzmann, but whatever the technique it remains a heavy task since there are four unknowns at each node (the three velocity components and the pressure) which necessitate the solution of four equations (the projection of Newton's law on each axis and mass conservation). By comparison, the Laplace equation is scalar with a single unknown at each node.

The objective of this work is to replace the Stokes equations by an elliptical equation with a space dependent permeability. More precisely, the local permeability k is supposed to be proportional to $(r-\alpha)^{2\alpha}$ where r is the distance of the voxel to the closest wall, and α a constant; k is zero in the solid phase. The elliptical equation is $\text{div}(k \text{ grad } p) = 0$. A macroscopic pressure gradient is assumed to be exerted on the medium and again the resulting velocity averaged over space yields a permeability K_L .

In order to validate this method, systematic calculations have been performed.

First, elementary shapes (plane channel, circular pipe, rectangular channels) were studied for which flow occurs along parallel lines in which case K_L is the arithmetic average of the k 's. K_L was calculated for various discretizations of the pore space and various values of α . For $\alpha=0.5$, the agreement with the exact analytical value of K_S is excellent for the plane and rectangular channels while it is only approximate for circular pipes.

Second, the permeability K_L of channels with sinusoidal walls was calculated and compared with analytical results and numerical ones provided by a Lattice Boltzmann algorithm. Generally speaking, the discrepancy does not exceed 25% when $\alpha=0.5$.

Third, the most important test was performed on two types of real media that were used for previous studies. A fracture network measured by FIB/SEM in a low permeability sandstone was used for that purpose; the two dimensionless permeabilities K_S and K_L are equal to $9.3 \cdot 10^{-3}$ and $8.5 \cdot 10^{-3}$. Similar calculations were performed on 256 samples of Fontainebleau sandstones and the agreement was in general excellent, except may be for very low permeabilities.

To conclude, the Laplace solver is significantly more stable than the lattice Boltzmann approach, uses less memory, and is significantly faster. Permeabilities are in excellent agreement over a wide range of porosities.