

Analysis of the pullback attractors of a low-order quasigeostrophic ocean model under periodic and aperiodic forcing

Stefano Pierini (1,2), Michael Ghil (3,4), and Mickael D. Chekroun (4)

(1) Università di Napoli Parthenope, Dipartimento di Scienze e Tecnologie, Napoli, Italy (stefano.pierini@uniparthenope.it),
(2) CoNISMa, Rome, Italy, (3) École Normale Supérieure and PSL Research University, Paris, France, (4) University of California, Los Angeles, USA

A low-order quasigeostrophic model captures several key features of intrinsic low-frequency variability of the oceans' wind-driven circulation. This double-gyre model is used here as a prototype of an unstable and nonlinear dynamical system with time-dependent forcing to explore basic features of climate change in the presence of natural variability [1,2]. The studies rely on the theoretical framework of nonautonomous dynamical systems and of their pullback attractors (PBAs), namely the time-dependent invariant sets that attract all trajectories initialized in the remote past. Ensemble simulations help us explore these PBAs.

The chaotic PBAs of the periodically forced model [1] are found to be cyclo-stationary and cyclo-ergodic. Two parameters are then introduced to analyze the topological structure of the PBAs as a function of the forcing period; their joint use allows one to identify four distinct forms of sensitivity to initial state that correspond to distinct system behaviors. The model's response to periodic forcing turns out to be, in most cases, very sensitive to the initial state.

The system is then forced by a synthetic aperiodic forcing [2]. The existence of a global PBA is rigorously demonstrated. We then assess the convergence of trajectories to this PBA by computing the probability density function (PDF) of trajectory localization in the model's phase space. A sensitivity analysis with respect to forcing amplitude shows that the global PBA experiences large modifications if the underlying autonomous system is dominated by small-amplitude limit cycles, while the changes are less dramatic in a regime characterized by large-amplitude relaxation oscillations. The dependence of the attracting sets on the choice of the ensemble of initial states is then analyzed. Two types of basins of attraction coexist for certain parameter ranges; they contain chaotic and nonchaotic trajectories, respectively. The statistics of the former does not depend on the initial states, whereas the trajectories in the latter converge to small portions of the global PBA. This complex behavior requires, therefore, separate PDFs for chaotic and nonchaotic trajectories.

[1] Pierini S., 2014. *J. Phys. Oceanogr.*, 44, 3245-3254.

[2] Pierini S., M. Ghil and M. D. Chekroun, 2016. *J. Climate*, 29, 4185-4202.