



A Half-Cycle Discrete Fourier Transform (HCDFT) for the Improved Identification of Periodic Features in Time-Series

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The well known Fourier orthogonality conditions which underpin the Fourier Series, Fourier Transform and Discrete Fourier Transform (DFT), apply to sinusoids with frequencies of integer numbers of cycles over the duration of the time-series (or signal) in question. Considered as a linear transformation, the DFT maps an N-point time-series (or other signal) onto a linearly independent set of N frequency components at $f = 0, 1, 2, \dots$ cycles over the total duration of the time-series.

However, and uniquely in terms of non-integer numbers of cycles, these orthogonality conditions also apply to integer-plus-half cycles over the duration. Thus, this provides a basis for a Half-Cycle Discrete Fourier Transform (HCDFT) which maps an N-point time-series (or other signal) onto a linearly independent set of N frequency components at $f = 0.5, 1.5, 2.5, \dots$ cycles over the total duration of the time-series. In identifying half-cycles, this novel variant of the DFT enables improved frequency discrimination when used in conjunction with the DFT.

Each transform (DFT, HCDFT) separately accounts for all the variance in an N-point time-series, enabling estimation of statistical effect-size and significance. However, the two sets of frequency components are not necessarily mutually orthogonal: although the two sets of frequency components can be interleaved to produce a combined spectrum with $2N$ frequency components at $f = 0, 0.5, 1, 1.5, \dots$ cycles over the total duration of the time-series, the two sets essentially need to be considered separately for amplitude-power-variance purposes.

The forward and inverse HCDFT pair will be presented and evaluated and the application illustrated using ca. 250-year time-series of solar/sunspot cycles.