

## Dynamic stability of rolling particles between elastic plates

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Mechanical behaviour of a large class of engineering materials from granular materials and interlocking structures to damaged rock and fragmented rock masses is controlled by rotations of the constituents leading to the necessity to introduce internal rotations and rotational degrees of freedom into the constitutive laws. An important (and underappreciated) factor in these cases is the shape of rotating elements as it imposes, in the presence of compressive stress, kinematic constraints that provide additional resistance to rotations. (Despite the universal acknowledgement that the particles in geomaterial and concrete are far from being spherical, the vast majority of models, from the Cosserat continuum to the very recent are still based on spherical or circular elements, e.g. [1]). In the presence of high compressive loading the combination of rotation and non-sphericity of the constituents produces qualitatively new effects: the effect of apparent negative stiffness and a possibility of instability [2].

In order to simplify the analysis we investigate the stability of interacting rotating particles is a sympathetic oscillator [3]. A modified model of the oscillating system, given their rods slide without friction relative to each other was proposed in [4]. An important property of the system is the possibility of bifurcations depending on the distance between the suspension points. Here we study the stability of two linearly – interacting oscillators in a uniform gravity field. The simplest models pertinent to the analysis of stability of such systems is a system of two linked oscillators (masses on rods) with a modification that allows the rods to intersect and slide without friction relative to each other thus providing a simplified modelling of 3D situations. We demonstrate that the analysis of the system trajectories is the most convenient in a 2D parametric space which is generated by the initial relative distance between the masses and the potential energy of the oscillator normalised by the spring's effective energy.

We found that the system has symmetrical and asymmetrical equilibrium solutions. In the dynamical case, the asymmetric solutions ensue when the system is deviated from its symmetrical equilibrium. We point out a few features of the global dynamics of the system: 1) at the symmetric equilibrium, the stability region of the sympathetic oscillators does not intersect the stability region of the modified oscillator system below the horizontal suspension line; 2) at the asymmetric equilibrium, the stability region of the sympathetic oscillators intersect the stability region of the modified oscillator system below the suspension line.

### References

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