Obtaining orthotropic elasticity tensor using entries zeroing method.

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A generally anisotropic elasticity tensor obtained from measurements can be represented by a tensor belonging to one of eight material symmetry classes. Knowledge of symmetry class and orientation is helpful for describing physical properties of a medium. For each non-trivial symmetry class except isotropic this problem is nonlinear. A common method of obtaining effective tensor is a choosing its non-trivial symmetry class and minimizing Frobenius norm between measured and effective tensor in the same coordinate system. Global optimization algorithm has to be used to determine the best rotation of a tensor. In this contribution, we propose a new approach to obtain optimal tensor, with the assumption that it is orthotropic (or at least has a similar shape to the orthotropic one). In orthotropic form tensor 24 out of 36 entries are zeros. The idea is to minimize the sum of squared entries which are supposed to be equal to zero through rotation calculated with optimization algorithm - in this case Particle Swarm Optimization (PSO) algorithm. Quaternions were used to parametrize rotations in 3D space to improve computational efficiency. In order to avoid a choice of local minima we apply PSO several times and only if we obtain similar results for the third time we consider it as a correct value and finish computations. To analyze obtained results Monte-Carlo method was used. After thousands of single runs of PSO optimization, we obtained values of quaternion parts and plot them. Points concentrate in several points of the graph following the regular pattern. It suggests the existence of more complex symmetry in the analyzed tensor. Then thousands of realizations of generally anisotropic tensor were generated – each tensor entry was replaced with a random value drawn from normal distribution having a mean equal to measured tensor entry and standard deviation of the measurement. Each of these tensors was subject of PSO based optimization delivering quaternion for optimal rotation. Computations were parallelized with OpenMP to decrease computational time what enables different tensors to be processed by different threads. As a result the distributions of rotated tensor entries values were obtained. For the entries which were to be zeroed we can observe almost normal distributions having mean equal to zero or sum of two normal distributions having inverse means. Non-zero entries represent different distributions with two or three maxima. Analysis of obtained results shows that described method produces consistent values of quaternions used to rotate tensors. Despite of less complex target function in a process of optimization in comparison to common approach, entries zeroing method provides results which can be applied to obtain an orthotropic tensor with good reliability. Modification of the method can produce also a tool for obtaining effective tensors belonging to another symmetry classes. This research was supported by the Polish National Science Center under contract No.\ DEC-2013/11/B/ST10/0472.