

A state-space representation of the GR4J rainfall-runoff model

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In hydrology, the majority of conceptual models are available only in discrete form. This means that the formulations of the models are based on discrete equations instead of continuous ordinary differential equations (ODE) (see Clark and Kavetski, 2010). The time-step is often “hardcoded” in the model formulation. This can represent a problem in particular for creating a time step-variable model.

Furthermore, the fluxes in the models are treated sequentially. For example, in the simple GR4J model, the precipitations (if any) are first added to the production store. Then, the updated level is used to compute the percolation from the store. The resulting level obtained at the end of the time step is different to the level which would be obtained if the two operations (i.e. addition of precipitation and percolation) were done simultaneously. Mathematically, this corresponds to an approximation of ODE solution which is called “operator splitting”. This allows to solve an equation even if finding an exact solution is impossible but the error produced by this approximation is difficult to determine. For this reason, it is not easy to separate the numerical error of the resolution from the conceptual error. It could represent an important issue to better understand model behaviour and to identify possible improvements.

The aim of this presentation is to detail a state-space representation of the simple GR4J model. The state-space representation aims to represent GR4J by an ODE system which provides the internal variables of the model at all times. We will present here the choices made to adapt GR4J to the state-space formulation and to numerically solve this system. Modifications of the model’s equations were also made to adapt the model to lower time step in case it would be used for a time step-variable application.

The results obtained with this state-space representation of GR4J were very similar to those of the original model in terms of performances and hydrographs.

This state-space representation could be useful for data assimilation methods in prediction (for example the use of Kalman filters) and for variable time step modelling. It also finds applications for dynamic multimodel combination (i.e. SUMO method, van den Berge et al., 2011) as shown in abstract EGU2017-4638.

References

- Clark, M. P. and Kavetski, D. (2010). Ancient numerical daemons of conceptual hydrological modeling : 1. fidelity and efficiency of time stepping schemes. *Water Resour. Res.*, 46(10).
- van den Berge, L. A., Selten, F. M., Wiegerinck, W., and Duane, G. S. (2011). A multi-model ensemble method that combines imperfect models through learning. *Earth System Dynamics*, 2(1) :161–177.