

## Stratified flows with variable density: mathematical modelling and numerical challenges.

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Stratified flows appear in a wide variety of fundamental problems in hydrological and geophysical sciences. They may involve from hyperconcentrated floods carrying sediment causing collapse, landslides and debris flows, to suspended material in turbidity currents where turbulence is a key process. Also, in stratified flows variable horizontal density is present. Depending on the case, density varies according to the volumetric concentration of different components or species that can represent transported or suspended materials or soluble substances. Multilayer approaches based on the shallow water equations provide suitable models but are not free from difficulties when moving to the numerical resolution of the governing equations. Considering the variety of temporal and spatial scales, transfer of mass and energy among layers may strongly differ from one case to another. As a consequence, in order to provide accurate solutions, very high order methods of proved quality are demanded. Under these complex scenarios it is necessary to observe that the numerical solution provides the expected order of accuracy but also converges to the physically based solution, which is not an easy task. To this purpose, this work will focus in the use of Energy balanced augmented solvers, in particular, the Augmented Roe Flux ADER scheme.

### References:

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