

Bayesian model selection: Evidence estimation based on DREAM simulation and bridge sampling

Elena Volpi (1), Gerrit Schoups (2), Giovanni Firmani (1), Jasper A. Vrugt (3,4)

(1) Univeristy Roma Tre, Department of Engineering, Rome, Italy (elena.volpi@uniroma3.it), (2) Department of Water Management, Delft University of Technology, the Netherlands, (3) Department of Civil and Environmental Engineering, University of California, Irvine, CA, USA, (4) Department of Earth System Science, University of California, Irvine, CA, USA

Bayesian inference has found widespread application in Earth and Environmental Systems Modeling, providing an effective tool for prediction, data assimilation, parameter estimation, uncertainty analysis and hypothesis testing. Under multiple competing hypotheses, the Bayesian approach also provides an attractive alternative to traditional information criteria (e.g. AIC, BIC) for model selection. The key variable for Bayesian model selection is the evidence (or marginal likelihood) that is the normalizing constant in the denominator of Bayes theorem; while it is fundamental for model selection, the evidence is not required for Bayesian inference. It is computed for each hypothesis (model) by averaging the likelihood function over the prior parameter distribution, rather than maximizing it as by information criteria; the larger a model evidence the more support it receives among a collection of hypothesis as the simulated values assign relatively high probability density to the observed data. Hence, the evidence naturally acts as an Occam's razor, preferring simpler and more constrained models against the selection of over-fitted ones by information criteria that incorporate only the likelihood maximum. Since it is not particularly easy to estimate the evidence in practice, Bayesian model selection via the marginal likelihood has not yet found mainstream use.

We illustrate here the properties of a new estimator of the Bayesian model evidence, which provides robust and unbiased estimates of the marginal likelihood; the method is coined Gaussian Mixture Importance Sampling (GMIS). GMIS uses multidimensional numerical integration of the posterior parameter distribution via bridge sampling (a generalization of importance sampling) of a mixture distribution fitted to samples of the posterior distribution derived from the DREAM algorithm (Vrugt et al., 2008; 2009). Some illustrative examples are presented to show the robustness and superiority of the GMIS estimator with respect to other commonly used approaches in the literature.