



## **Variational data assimilation for limited-area models: solution of the open boundary control problem and its application for the Gulf of Finland**

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The problem of modeling water areas with ‘liquid’ (open) lateral boundaries is discussed. There are different known methods dealing with open boundaries in limited-area models, and one of the most efficient is data assimilation. Although this method is popular, there are not so many articles concerning its implementation for recovering boundary functions. However, the problem of specifying boundary conditions at the open boundary of a limited area is still actual and important.

The mathematical model of the Baltic Sea circulation, developed in INM RAS, is considered. It is based on the system of thermo–hydrodynamic equations in the Boussinesq and hydrostatic approximations. The splitting method that is used for time approximation in the model allows to consider the data assimilation problem as a sequence of linear problems. One of such ‘simple’ temperature (salinity) assimilation problem is investigated in the study. Using well known techniques of study and solution of inverse problems and optimal control problems [1], we propose an iterative solution algorithm and we obtain conditions for existence of the solution, for unique and dense solvability of the problem and for convergence of the iterative algorithm. The investigation shows that if observations satisfy certain conditions, the proposed algorithm converges to the solution of the boundary control problem. Particularly, it converges when observational data are given on the ‘liquid’ boundary [2].

Theoretical results are confirmed by the results of numerical experiments. The numerical algorithm was implemented to water area of the Baltic Sea. Two numerical experiments were carried out in the Gulf of Finland: one with the application of the assimilation procedure and the other without. The analyses have shown that the surface temperature field in the first experiment is close to the observed one, while the result of the second experiment misfits. Number of iterations depends on the regularisation parameter, but generally the algorithm converges after 10 iterations. The results of the numerical experiments show that the usage of the proposed method makes sense.

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