

## Comparing and improving proper orthogonal decomposition (POD) to reduce the complexity of groundwater models

Moritz Gosses (1), Wolfgang Nowak (2), Thomas Wöhling (1,3)

(1) Technische Universität Dresden, Institute of Hydrology and Meteorology, Department of Hydrology, Dresden, Germany (moritz.gosses@tu-dresden.de), (2) Universität Stuttgart, Institute for Modelling Hydraulic and Environmental Systems (LH2)/SimTech, Stuttgart, Germany, (3) Lincoln Agritech Ltd, Christchurch & Hamilton, New Zealand

Physically-based modeling is a wide-spread tool in understanding and management of natural systems. With the high complexity of many such models and the huge amount of model runs necessary for parameter estimation and uncertainty analysis, overall run times can be prohibitively long even on modern computer systems. An encouraging strategy to tackle this problem are model reduction methods.

In this contribution, we compare different proper orthogonal decomposition (POD, Siade et al. (2010)) methods and their potential applications to groundwater models. The POD method performs a singular value decomposition on system states as simulated by the complex (e.g., PDE-based) groundwater model taken at several time-steps, so-called snapshots. The singular vectors with the highest information content resulting from this decomposition are then used as a basis for projection of the system of model equations onto a subspace of much lower dimensionality than the original complex model, thereby greatly reducing complexity and accelerating run times. In its original form, this method is only applicable to linear problems. Many real-world groundwater models are non-linear, tough. These non-linearities are introduced either through model structure (unconfined aquifers) or boundary conditions (certain Cauchy boundaries, like rivers with variable connection to the groundwater table). To date, applications of POD focused on groundwater models simulating pumping tests in confined aquifers with constant head boundaries. In contrast, POD model reduction either greatly loses accuracy or does not significantly reduce model run time if the above-mentioned non-linearities are introduced. We have also found that variable Dirichlet boundaries are problematic for POD model reduction.

An extension to the POD method, called POD-DEIM, has been developed for non-linear groundwater models by Stanko et al. (2016). This method uses spatial interpolation points to build the equation system in the reduced model space, thereby allowing the recalculation of system matrices at every time-step necessary for non-linear models while retaining the speed of the reduced model. This makes POD-DEIM applicable for groundwater models simulating unconfined aquifers. However, in our analysis, the method struggled to reproduce variable river boundaries accurately and gave no advantage for variable Dirichlet boundaries compared to the original POD method.

We have developed another extension for POD that targets to address these remaining problems by performing a second POD operation on the model matrix on the left-hand side of the equation. The method aims to at least reproduce the accuracy of the other methods where they are applicable while outperforming them for setups with changing river boundaries or variable Dirichlet boundaries. We compared the new extension with original POD and POD-DEIM for different combinations of model structures and boundary conditions. The new method shows the potential of POD extensions for applications to non-linear groundwater systems and complex boundary conditions that go beyond the current, relatively limited range of applications.

### References:

- Siade, A. J., Putti, M., and Yeh, W. W.-G. (2010). Snapshot selection for groundwater model reduction using proper orthogonal decomposition. *Water Resour. Res.*, 46(8):W08539.
- Stanko, Z. P., Boyce, S. E., and Yeh, W. W.-G. (2016). Nonlinear model reduction of unconfined groundwater flow using pod and deim. *Advances in Water Resources*, 97:130 - 143.