

## Differential embedding – From theory to application in palaeoclimatology

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Analysing data from palaeoclimate archives such as tree rings or lake sediments offers the opportunity of inferring information on past climate variability. Often, such data sets are univariate and a proper reconstruction of the system's higher dimensional phase space can be crucial for further analyses. In this study, we revisit the concept of differential embedding as an alternative approach to time-delay embedding and aim at bringing it closer to application.

Differential embedding relates the system's higher dimensional coordinates to the derivatives of the measured time series. For implementation, this requires robust and efficient algorithms to estimate derivatives from possibly irregularly sampled data. For this purpose, we consider several approaches: (i) central differences adapted to irregular sampling, (ii) a generalised version of discrete Legendre coordinates [1] and (iii) the concept of Moving Taylor Bayesian Regression [2]. We then systematically evaluate the performance of differential and time-delay embedding by comparing geometrical properties of the embedded attractors to those of the original attractor for two paradigmatic model systems – the Rössler and the Lorenz system. The quality of the phase space reconstruction is quantified using recurrence network analysis.

Our results suggest that differential embedding can indeed be an alternative to time-delay embedding for unequally spaced time series. Hence, using differential embedding and recurrence network methods to analyse palaeoclimate data sets opens up good prospects of gaining new insights into past climate variability. We demonstrate the potential of this approach for real-world applications by applying derivative embedding in combination with recurrence network analysis to some well-studied palaeoclimate data sets from South America.

[1] J. F. Gibson, J. D. Farmer, M. Casdagli, and S. Eubank. An analytic approach to practical phase space reconstruction. *Physica*, 57D(1), 1992.

[2] J. Heitzig. Moving Taylor Bayesian Regression for nonparametric multidimensional function estimation with possibly correlated errors. arXiv:1204.2841, 2012.