



## **Non-linear Interactions of Rossby waves in shallow water magnetohydrodynamics on a beta-plane**

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Rotating magnetohydrodynamic shallow water equations are obtained from conventional magnetohydrodynamic equations for incompressible inviscid heavy plasma layer with free surface in an external vertical magnetic field. The pressure is assumed to be hydrostatic, and the water layer height is considered to be much smaller than horizontal scales. The MHD shallow water equations with an external vertical magnetic field are revised by supplementing them with the equations that are consequences of the magnetic field divergence-free conditions and reveal the existence of a third component of the magnetic field in such approximation providing its relation with the horizontal magnetic field. It is shown that the presence of a vertical magnetic field significantly changes the dynamics of the wave processes in astrophysical plasma compared to the neutral fluid and plasma layer in a horizontal magnetic field. We have investigated the interaction of wave packets in the magnetohydrodynamic shallow water flows in an external vertical magnetic field and in horizontal (toroidal and poloidal) magnetic fields on a  $\beta$ -plane. Linear analysis leads to magneto-Rossby waves in  $\beta$ -plane approximation. In the absence of the horizontal magnetic field the dynamics of plasma appears to be similar to the neutral fluid dynamics. Using the asymptotic multiscale method we obtained the non-linear interaction equations for the waves amplitudes. The analysis of the amplitudes equations shows that on  $\beta$ -plane there are two types of instabilities: one magneto-Rossby wave decays into two magneto-Rossby waves and a magneto-Rossby wave amplifies in the field of two magneto-Rossby waves. These instabilities occur in both cases: in the external vertical magnetic field and in the horizontal (toroidal and poloidal) magnetic field. For all types of instabilities the growth rates are found.