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# A structure-preserving split finite element discretization of the split 1D linear shallow-water equations

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We present a locally conservative, low-order finite element (FE) discretization of the covariant 1D linear shallow-water equations written in split form (cf. [1]). The introduction of additional differential forms (DF) that build pairs with the original ones permits a splitting of these equations into topological momentum and continuity equations and metric-dependent closure equations that apply the Hodge-star. Our novel discretization framework conserves this geometrical structure, in particular it provides for all DFs proper FE spaces such that the differential operators (here gradient and divergence) hold in strong form. The discrete topological equations simply follow by trivial projections onto piecewise constant FE spaces without need to partially integrate. The discrete Hodge-stars operators, representing the discretized metric equations, are realized by nontrivial Galerkin projections (GP). Here they follow by projections onto either a piecewise constant (GP0) or a piecewise linear (GP1) space.

Our framework thus provides essentially three different schemes with significantly different behavior. The split scheme using twice GP1 is unstable and shares the same discrete dispersion relation and similar second-order convergence rates as the conventional P1-P1 FE scheme that approximates both velocity and height variables by piecewise linear spaces. The split scheme that applies both GP1 and GP0 is stable and shares the dispersion relation of the conventional P1-P0 FE scheme that approximates the velocity by a piecewise linear and the height by a piecewise constant space with corresponding second- and first-order convergence rates. Exhibiting for both velocity and height fields second-order convergence rates, we might consider the split GP1-GP0 scheme though as stable versions of the conventional P1-P1 FE scheme. For the split scheme applying twice GP0, we are not aware of a corresponding conventional formulation to compare with. Though exhibiting larger absolute error values, it shows similar convergence rates as the other split schemes, but does not provide a satisfactory approximation of the dispersion relation as short waves are propagated much to fast. Despite this, the finding of this new scheme illustrates the potential of our discretization framework as a toolbox to find and to study new FE schemes based on new combinations of FE spaces.

## References

[1] Bauer, W. [2016], A new hierarchically-structured n-dimensional covariant form of rotating equations of geophysical fluid dynamics, *GEM - International Journal on Geomathematics*, **7(1)**, 31–101.