European Geosciences Union (EGU)
General Assembly
23 – 28 April 2017, Vienna - Austria

Earth gravity field modeling and relativistic measurements with laser-ranged satellites and the LARASE research program

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Summary

- The LARASE experiment and its goals
- Relativistic effects to be measured
- Systematic errors from the background gravitational field
- Some recent measurements of relativistic effects
- Conclusions and future work

The LARASE experiment and its goals

The LARASE goals:

- The **LAser RAnged Satellites Experiment** (**LARASE**) main goal is to provide accurate measurements for the gravitational interaction in the **weak-field** and **slow-motion** limit of **General Relativity** by means of a very precise laser tracking of geodetic satellites orbiting around the Earth (the two **LAGEOS** and **LARES**)
- Beside the quality of the tracking observations, guaranteed by the powerful **Satellite Laser Ranging** (**SLR**) technique of the **International Laser Ranging Service** (**ILRS**), also the quality of the dynamical models implemented in the **Precise Orbit Determination** (**POD**) software plays a fundamental role in order to obtain precise and accurate measurements
- The models have to account for the perturbations due to both gravitational and non-gravitational forces in such a way to reduce as much as possible the difference between the *observed* range, from the tracking, and the *computed* one, from the models
- In particular, **LARASE** aims to improve the dynamical models of the current best laser-ranged satellites in order to perform a precise and accurate orbit determination, able to benefit also space geodesy and geophysics

The LARASE experiment and its goals

The LARASE activities:

1. Review of the literature, technical notes and all the documentation (**NASA**, **ALENIA**, **ASI**) related with the structure of the satellites and their physical characteristics
2. A reconstruction of the internal and external structure of the satellites with finite elements techniques
3. Review of the spin model of the two **LAGEOS** satellites and of their complex interaction with the Earth's magnetic field
4. Develop a spin model for **LARES**
5. Extension of the Yarkovsky–Schach thermal effect to the low spin-rate approximation
6. Impact of the neutral drag on the two **LAGEOS** satellites and on **LARES**
7. Solid and Ocean tides on the two **LAGEOS** satellites and on **LARES**
8. Precise Orbit Determination for the two **LAGEOS** satellites and for **LARES**
9. Finite Element Model for the Thermal effects

LAser **RE**lativity **S**atellite

The LARASE experiment and its goals

LAGEOS, LAGEOS II and LARES

orbit, size, mass and materials

LAser **GEO**dynamic **S**atellite
LAGEOS II

LAGEOS (NASA 1976)
LAGEOS II (NASA/ASI 1992)
LARES (ASI 2012)

Parameter		LARES	LAGEOS	LAGEOS II
a	[km]	7 820	12 270	12 163
e		0.001	0.004	0.014
I	[deg]	69.5	109.8	52.7
R	[cm]	18.2	30	30
M	[kg]	386.8	406.9	405.4
A/M	[m2/kg]	$2.69 \cdot 10^{-4}$	$6.94 \cdot 10^{-4}$	$6.97 \cdot 10^{-4}$

$$\left.\frac{A}{M}\right|_{Lares} \cong \frac{1}{2.6}\left.\frac{A}{M}\right|_{Lageos}$$

	LARES	LAGEOS
material	Tungsten	Al/Brass/Be/Cu
CCR (suprasil 311)	92	422 + 4
bin	30 s	120 s

The LARASE experiment and its goals

- Despite the smaller A/M ratio, the non-gravitational accelerations are not always smaller in magnitude for LARES with respect to LAGEOS II (or LAGEOS), due to the lower height (1450 vs. 5900 km) and the higher density of neutral atmosphere
- Being 50 times larger on LARES than on the two LAGEOS, the accurate modeling of neutral atmosphere drag needs special attention, because it might mask the presence of smaller and subtler effects

Effect	Estimate	LAGEOS II	LARES
Earth's monopole	$\frac{GM_\oplus}{r^2}$	2.69	6.51
Earth's oblateness	$3\frac{GM_\oplus}{r^2}\left(\frac{R_\oplus}{r}\right)^2\bar{C}_{2,0}$	-1.1×10^{-3}	-6.4×10^{-3}
Low-order geopotential harmonics	$3\frac{GM_\oplus}{r^2}\left(\frac{R_\oplus}{r}\right)^2\bar{C}_{2,2}$	5.4×10^{-6}	3.2×10^{-5}
High-order geopotential harmonics	$19\frac{GM_\oplus}{r^2}\left(\frac{R_\oplus}{r}\right)^{18}\bar{C}_{18,18}$	1.4×10^{-12}	4.6×10^{-9}
Moon perturbation	$2\frac{GM_☾}{r_☾^3}r$	2.2×10^{-6}	1.4×10^{-6}
Sun perturbation	$2\frac{GM_\odot}{r_\odot^3}r$	9.6×10^{-7}	6.2×10^{-7}
General relativistic correction	$\frac{GM_\oplus}{r^2}\frac{GM_\oplus}{c^2}\frac{1}{r}$	9.8×10^{-10}	3.7×10^{-9}
Atmospheric drag	$\frac{1}{2}C_D\frac{A}{M}\rho V^2$	-2.6×10^{-13}	-1.3×10^{-11}
Solar radiation pressure	$C_R\frac{A}{M}\frac{\Phi_\odot}{c}$	3.2×10^{-9}	1.2×10^{-9}
Albedo radiation pressure	$C_R\frac{A}{M}\frac{\Phi_\odot}{c}A_\oplus\left(\frac{R_\oplus}{r}\right)^2$	3.5×10^{-10}	2.4×10^{-10}
Thermal emission	$\frac{4}{9}\frac{A}{M}\frac{\Phi_\odot}{c}\alpha\frac{\Delta T}{T_0}$	2.8×10^{-11}	not available
Dynamic solid tide	$3k_2\frac{GM_☾}{r_☾}\left(\frac{R_\oplus}{r_☾}\right)^2\frac{R_\oplus^3}{r^4}$	3.7×10^{-6}	2.2×10^{-5}
Dynamic ocean tide	~ 0.1 of the dynamic solid tide	3.7×10^{-7}	2.2×10^{-6}

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Relativistic effects to be measured

Our main goals in the field of fundamental physics measurements fall in the following main targets:

- Schwarzschild precession (gravitoelectric field)
- Lense-Thirring precession (gravitomagnetic field)
- Geodetic (de Sitter) precession
- Post-Newtonian parameter (β, γ, $\alpha 1$, $\alpha 2$, …)
- Constraints and limits to alternative theories of the gravitational interaction (Yukawa, non-symmetric/torsional …)

We are now ready to start new refined measurements of the above relativistic effects with laser-ranged satellites. As said, there are two main aspects to satisfy:

1. obtain very precise measurements from the analysis of the post-fit residuals (after the POD)
2. provide a very reliable estimate of the systematics, i.e., accurate measurements

Relativistic effects to be measured

Gravito-electromagnetism: linearized theory of General Relativity (GR)

In the Weak-Field and Slow-Motion (WFSM) limit of the theory of GR, Einstein's equations reduce to a form quite similar to those of electromagnetism. Following this approach we have a:

- gravitoelectric field produced by masses, analogous to the electric field produced by charges
- gravitomagnetic field produced by mass currents, analogous to the magnetic field produced by electric currents.

$$G_{\alpha\beta} = 8\pi \frac{G}{c^4} T_{\alpha\beta}$$

$$\eta_{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\begin{cases} \bar{h}^{\alpha\beta}{}_{,\beta} = 0 \\ g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} \\ \Delta \bar{h}_{\alpha\beta} = 16\pi \frac{G}{c^4} T_{\alpha\beta} \end{cases}$$

$$\begin{cases} \bar{h}_{\alpha\beta} \equiv h_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta} h \\ h \equiv h^{\alpha}_{\alpha} = \eta^{\alpha\beta} h_{\alpha\beta} \end{cases}$$

$$\left| h_{\alpha\beta} \right| \cong \left| \frac{\Phi}{c^2} \right| \le 10^{-6}$$

$$\begin{cases} \bar{h}^{00} = 4\frac{\Phi}{c^2} \\ \bar{h}^{0l} = -2\frac{A^l}{c^2} \\ \bar{h}^{ij} = O\left(c^{-4}\right) \end{cases}$$

$$\Phi = -\frac{GM_\odot}{R_\odot}$$

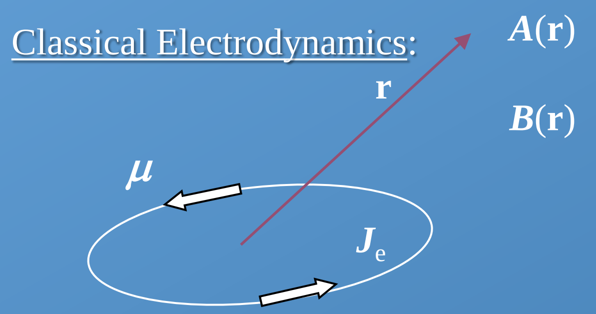
Gravitoelectric potential

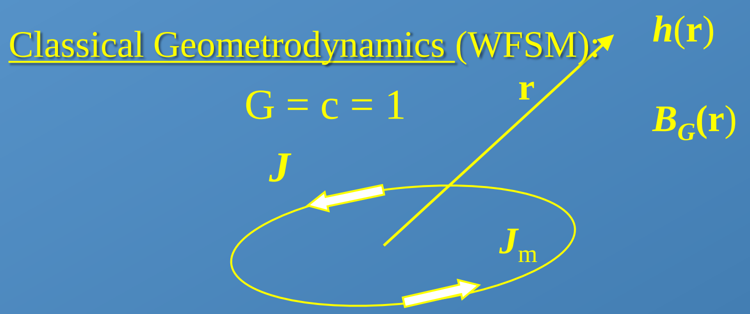
$$A^l = \frac{G}{c}\frac{J^n x^k}{r^3}\varepsilon^l_{nk}$$

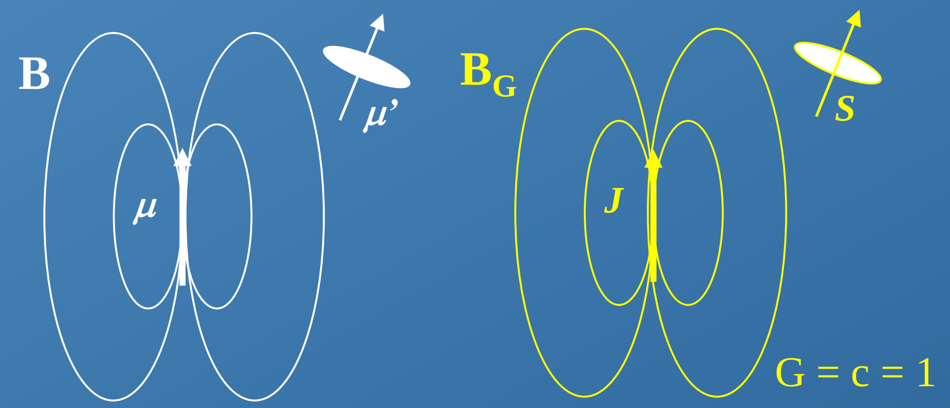
Gravitomagnetic potential

Relativistic effects to be measured

Formal analogy with electrodynamics: linearized theory of General Relativity (WFSM limit)







$$\Delta \vec{A} = -4\pi \cdot \vec{J}_e \qquad\qquad \Delta \vec{h} = 16\pi \cdot \vec{J}_m$$

solution:

$$\vec{A}(\vec{r}) = \int \frac{\vec{J}_e(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} d^3r' \qquad\qquad \vec{h}(\vec{r}) = -4\int \frac{\vec{J}_m(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} d^3r'$$

$$\vec{\mu} = \frac{1}{2}\int \vec{r} \wedge \vec{J}_e(\vec{r})\, d^3r \qquad\qquad \vec{J} = \int \vec{r} \wedge \vec{J}_m(\vec{r})\, d^3r$$

$$\vec{A}(\vec{r}) \cong \frac{\vec{\mu} \wedge \vec{r}}{r^3} \qquad\qquad \vec{h}(\vec{r}) \cong -2\frac{\vec{J} \wedge \vec{r}}{r^3}$$

$$\vec{B} = \vec{\nabla} \wedge \vec{A} \cong \frac{3\hat{r}(\hat{r} \bullet \vec{\mu}) - \vec{\mu}}{r^3} \qquad\qquad \vec{B}_G = \vec{\nabla} \wedge \vec{h} \cong -2\frac{3\hat{r}(\hat{r} \bullet \vec{J}) - \vec{J}}{r^3}$$

$$\boxed{\vec{F} = m\ddot{\vec{r}} = q(\vec{E} + \dot{\vec{r}} \wedge \vec{B})} \qquad\qquad \boxed{\vec{F} = m\ddot{\vec{r}} = m(-\frac{M}{r^2}\hat{r} + \dot{\vec{r}} \wedge \vec{B}_G)}$$

$$\vec{F} = (\vec{\mu}\,' \bullet \vec{\nabla})\vec{B} \qquad\qquad \vec{F} = \frac{1}{2}(\vec{S} \bullet \vec{\nabla})\vec{B}_G$$

$$\vec{N} = \vec{\mu}\,' \wedge \vec{B} \qquad\qquad \vec{N} = \frac{1}{2}\vec{S} \wedge \vec{B}_G$$

$$\dot{\vec{\Omega}} = -\vec{B} = \frac{\vec{\mu} - 3\hat{r}(\hat{r} \bullet \vec{\mu})}{r^3} \qquad\qquad \dot{\vec{\Omega}} = -\frac{1}{2}\vec{B}_G = \frac{-\vec{J} + 3\hat{r}(\hat{r} \bullet \vec{J})}{r^3}$$

This phenomenon is known as dragging of gyroscopes or dragging of inertial frames

Therefore, mass currents (as the rotating Earth) drag gyroscopes and change the orientation of their axes

Relativistic effects to be measured

Gravitomagnetism

- Mass currents contribute to the curvature of spacetime
- Gravitomagnetism may be thought of as a manifestation of the way inertia originates in Einstein geometrodynamics … “inertia here arises from mass there” …
- The dragging of inertial frames or Lense-Thirring effect represents a weak manifestation (within GR) of Mach’s Principle (the experimental proof of the origin of local inertial forces, interpreted as gravitational forces)
- The full inclusion of Mach Principle in GR is still debated …
- Anyway, the astrophysical and cosmological consequences are very significant …

See ‘’Gravitation and Inertia’’, Ciufolini and Wheeler, 1995 for a deep insight into gravitomgnetism

Relativistic effects to be measured

Model for GR

Huang et al., Celest. Mech. & Dyn. Astron. 48, 1990

Relativistic perturbations

$$\vec{A}_E = \frac{Gm_\oplus}{c^2 r^3}\left[\left(4\frac{Gm_\oplus}{r} - v^2\right)\vec{r} + 4(\vec{r} * \vec{v})\vec{v}\right]$$

Einstein or Schwarzschild component

$$\vec{A}_{dS} = 2(\vec{\Omega} \wedge \vec{v})$$

De Sitter (or geodetic) component

$$\vec{A}_{LT} = 2\frac{Gm_\oplus}{c^2 r^3}\left[\frac{3}{r^2}(\vec{r} \wedge \vec{v})(\vec{r} * \vec{J}) + (\vec{v} \wedge \vec{J})\right]$$

Lense–Thirring component

with:

$$\vec{\Omega} \cong \frac{3}{2}(\vec{V}_E - \vec{V}_S) \wedge \left(-\frac{GM_S}{c^2 R_{ES}^3}\right)\vec{X}_{ES}$$

Where, capital letters refer to position, velocity, acceleration and mass in the barycentric reference frame, while small letters refer to the same quantities in the non–inertial geocentric reference system (E=Earth, S=Sun)

Relativistic effects to be measured

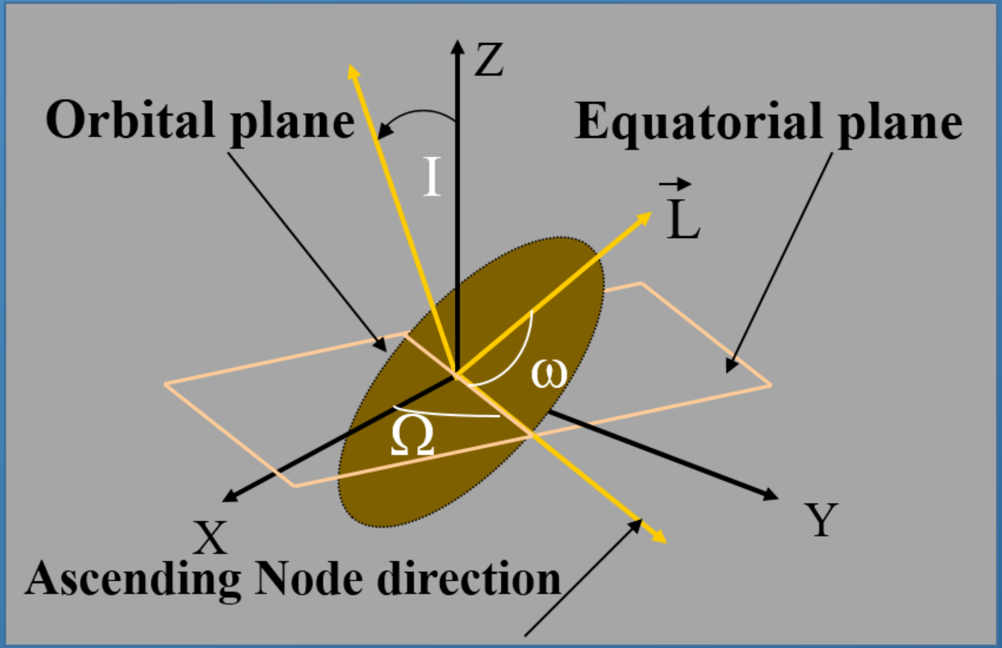
Gravitomagnetism: orbit precession

Lense-Thirring, Phys. Z, 19, 1918

$$\dot{\Omega}_{LT} = \mu \frac{2G}{c^2 a^3} \frac{J_\oplus}{(1-e^2)^{3/2}}$$

$$\dot{\omega}_{LT} = -\mu \frac{6G}{c^2 a^3} \frac{J_\oplus}{(1-e^2)^{3/2}} \cos I$$

$$\mu \neq \frac{1+\gamma}{2}$$



mas/yr	LAGEOS	LAGEOS II	LARES
$\dot{\Omega}_{LT}$	30.7	31.5	118.5
$\dot{\omega}_{LT}$	31.2	– 57.3	– 124.5

30 mas/yr at LAGEOS altitude (≅5900 km) corresponds to a displacement of about 1.8 m/yr!

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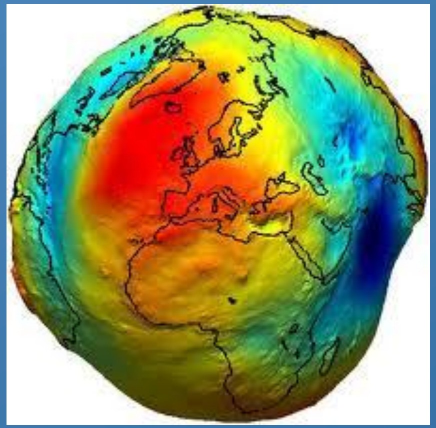
LARASE
LASER RANGED SATELLITES EXPERIMENT

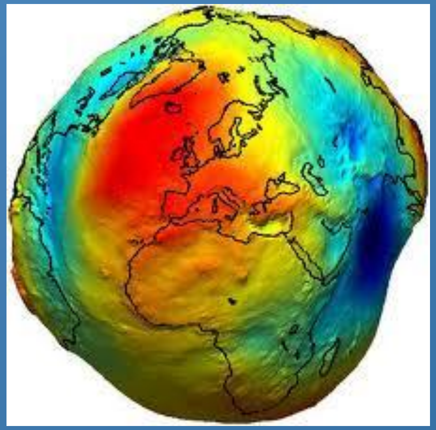
Systematics errors from the background gravitational field



In order to reach a precise and accurate measurement of a relativistic effect we need to perform a:

- precise orbit determination (POD) of the orbit of the satellites
  - observations (SLR data)
  - dynamical model (software)
- careful (robust) evaluation of the main systematic error sources
  - gravitational and non-gravitational
  - ad hoc analyses of the errors (formal and calibrated) of the various models considered

Systematics errors from the background gravitational field



Big problem with the **even zonal** harmonics uncertainties: systematic errors

$$V(r,\varphi,\lambda) = -\frac{GM_\oplus}{r}\left[1+\sum_{\ell=2}^{\infty}\sum_{m=0}^{\ell}\left(\frac{R_\oplus}{r}\right)^{\ell} P_{\ell m}(\sin\varphi)(C_{\ell m}\cos m\lambda + S_{\ell m}\sin m\lambda)\right]$$

Spherical harmonics development of the Earth's potential V(r)

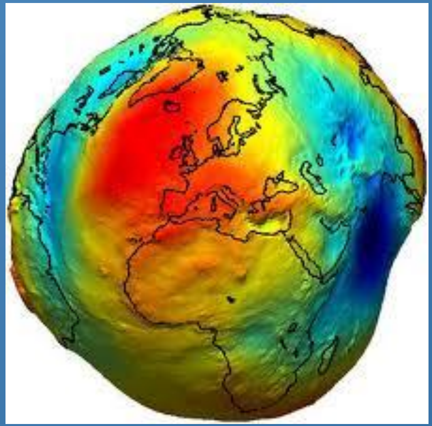
***m* = 0** $\rightarrow$ zonal harmonics

$$V(r) = -\frac{GM_\oplus}{r}\left[1 - J_2\left(\frac{R_\oplus}{r}\right)^2 \frac{3\cos^2\vartheta - 1}{2} + \cdots\right]$$

Dependency from the even zonal harmonics only, their uncertainties mimics a secular effect in the right ascension of the node and also in the argument of pericenter

$$\delta\dot{\Omega}_{class} = -\frac{3}{2} n \left(\frac{R_\oplus}{a}\right)^2 \frac{\cos i}{(1-e^2)^2}\delta J_2 + \cdots$$

Systematics errors from the background gravitational field



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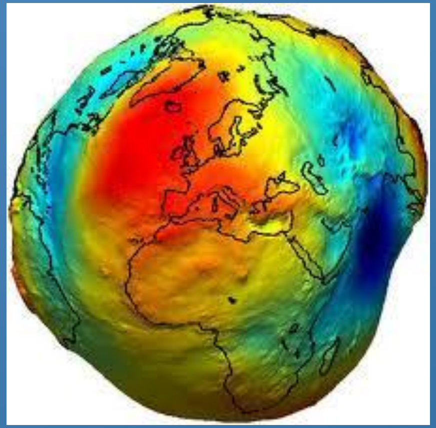
gument of pericenter

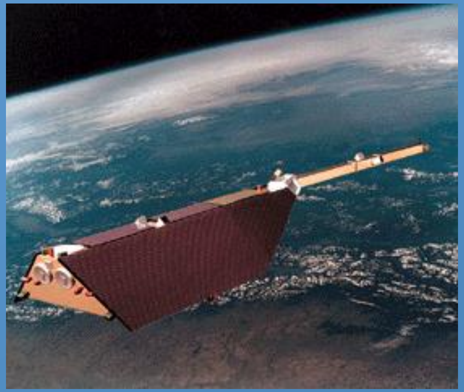
$$\dot{\Omega}^{Class} \cong -\frac{3}{2}n\left(\frac{R_\oplus}{a}\right)^2 \frac{\cos I}{\left(1-e^2\right)^2}\left\{J_2 + J_4\left[\frac{5}{8}\left(\frac{R_\oplus}{a}\right)^2\left(7\sin^2 I - 4\right)\frac{\left(1+\frac{3}{2}e^2\right)}{\left(1-e^2\right)^2}\right] + \ldots\right\}$$

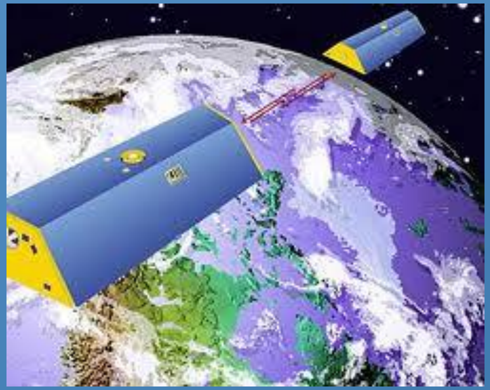
$$\dot{\omega}^{Class} \cong -\frac{3}{4}n\left(\frac{R_\oplus}{a}\right)^2 \frac{\left(1-5\cos^2 I\right)}{\left(1-e^2\right)^2}\left\{J_2 + J_4\left[\frac{5}{256}\left(\frac{R_\oplus}{a}\right)^2\left(7\sin^2 I - 4\right)\frac{C(e,I)}{\left(1-5\cos^2 I\right)}\right] + \ldots\right\}$$

Systematics errors from the background gravitational field



The **CHAMP**, **GRACE** and **GOCE** missions: high tech. space missions





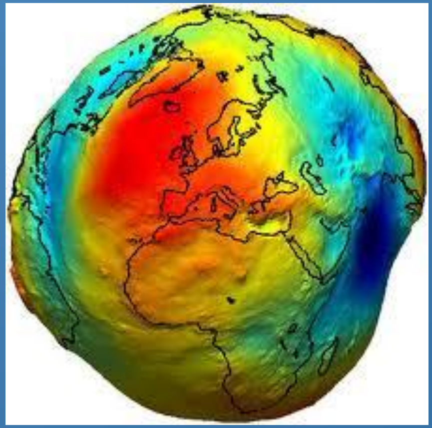
CHAMP carries:

1. GPS receiver for precise positioning, atmospheric profiling and bistatic altimetry;
2. STAR accelerometer for non–gravitational accelerations measurement
3. Advanced Stellar Compass for attitude control
4. magnetometers to measure the Earth's magnetic field;
5. laser retro–reflector array enabling laser ranging

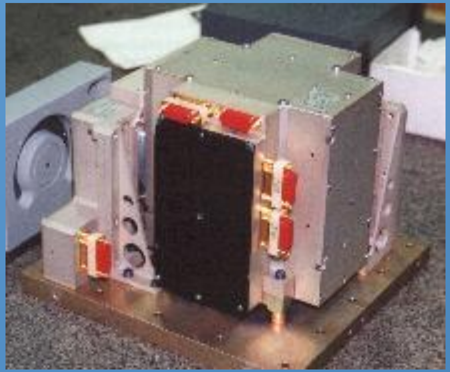
GRACE carries:

1. Black–Jack GPS receiver for precise positioning
2. SuperStar accelerometer for non–gravitational accelerations measurement;
3. Star Tracker (ST) for attitude control;
4. K–Band Ranging (KBR) system;
5. laser retro–reflector array enabling laser ranging

Systematics errors from the background gravitational field



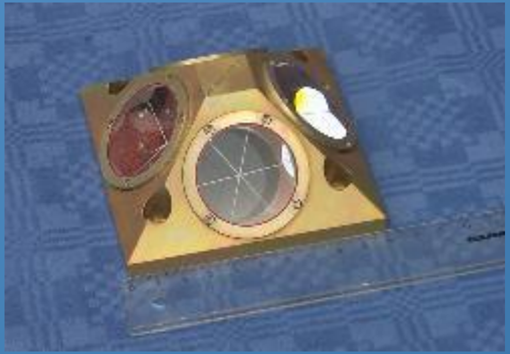
The **CHAMP**, **GRACE** and **GOCE** missions: high tech. space missions



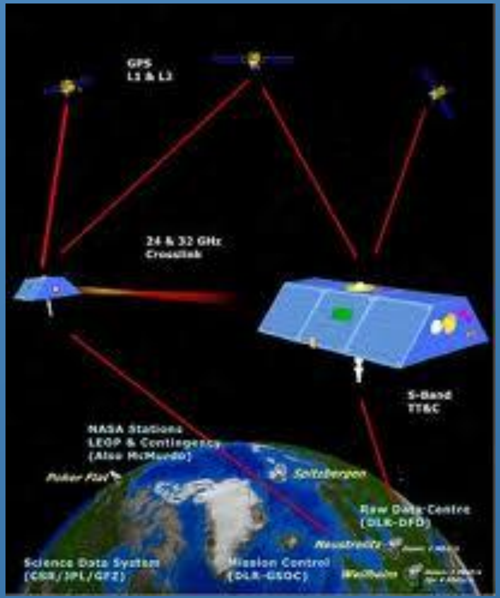
STAR accelerometer
(ONERA/CNES, France)

$$a \cong 10^{-9}\, m/s^2 \big/ \sqrt{Hz} \qquad \left(10^{-1} - 2\cdot 10^{-4}\right) Hz$$

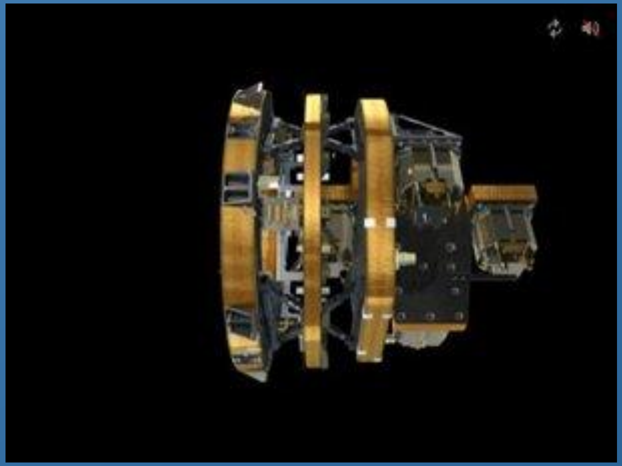
$$a \cong 10^{-10}\, m/s^2 \big/ \sqrt{Hz} \qquad \left(10^{-1} - 2\cdot 10^{-4}\right) Hz$$



LASER Retroreflector



Multiple tracking

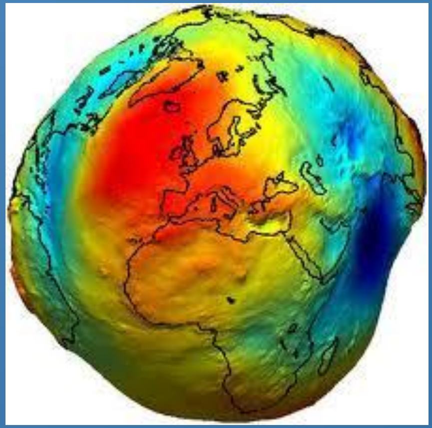


Gradiometer (EGG)

$$(10^{-1} - 5 \cdot 10^{-3}) Hz$$

$$a \cong (10^{-12} - 10^{-10})\, m/s^2/\sqrt{Hz}$$

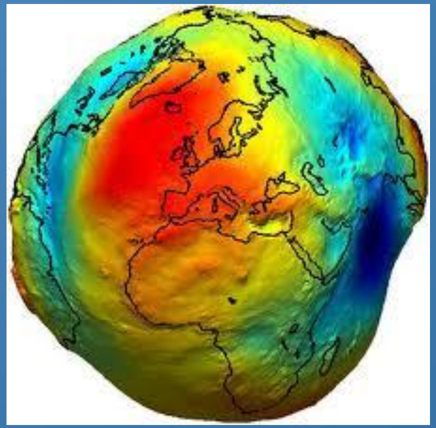
Systematics errors from the background gravitational field



The **CHAMP**, **GRACE** and **GOCE** missions

- huge improvement in the medium-wavelength and long-wavelength of the global gravity field of the Earth
- but no significant improvements in the low degree coefficients, since the low altitude of the involved satellites (~ 500 km and ~ 260 km)
- conversely, LAGEOS satellites are very sensitive to these low degree coefficients because of their much higher height (~ 6000 km)
- SLR data are used to compute the low degree coefficients and their time variation (also including data from the two LAGEOS)

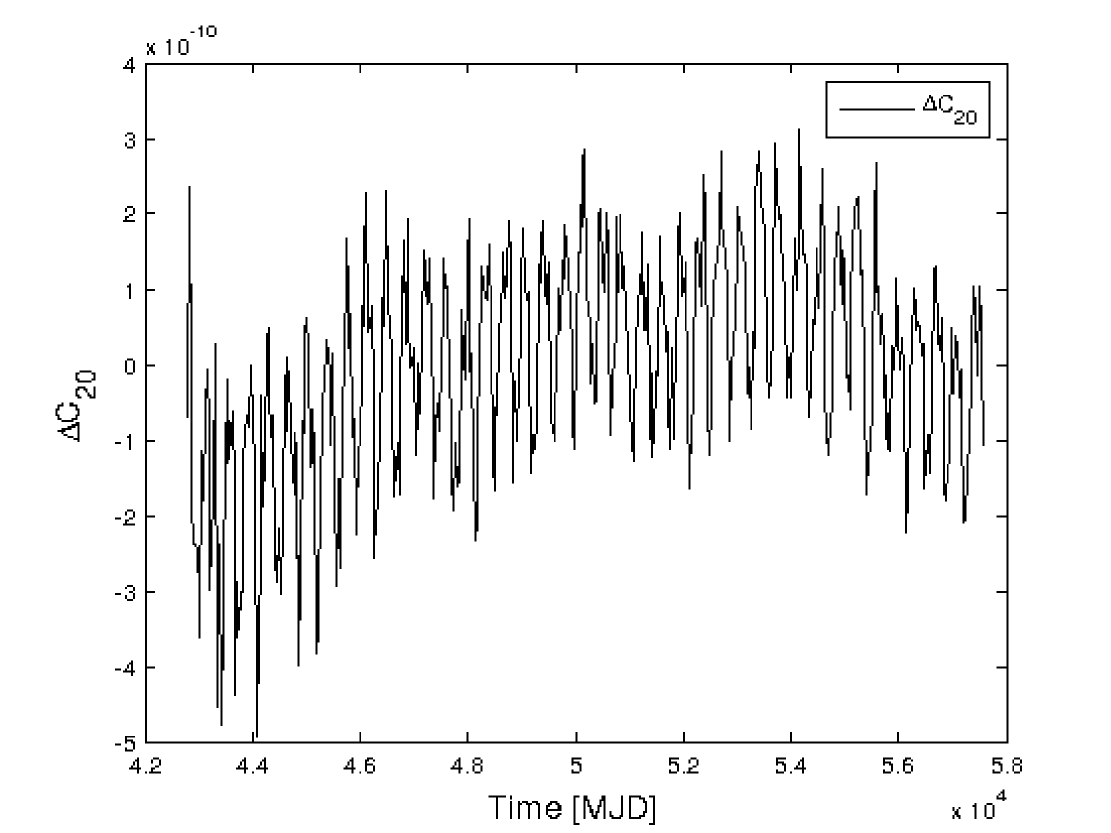
Systematics errors from the background gravitational field



The Earth's quadrupole coefficient C20: UT/CSR monthly time series (SLR)

- SLR data from 8 satellites from 1976 up to 2016.5 (corrected to the tide-free system)
- $\Delta C_{20} = C_{20} - \langle C_{20} \rangle$

Cheng et al., Deceleration in Earth's oblateness. JGR, 118 (2013)



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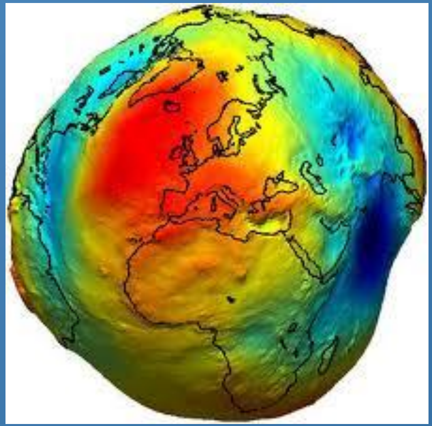

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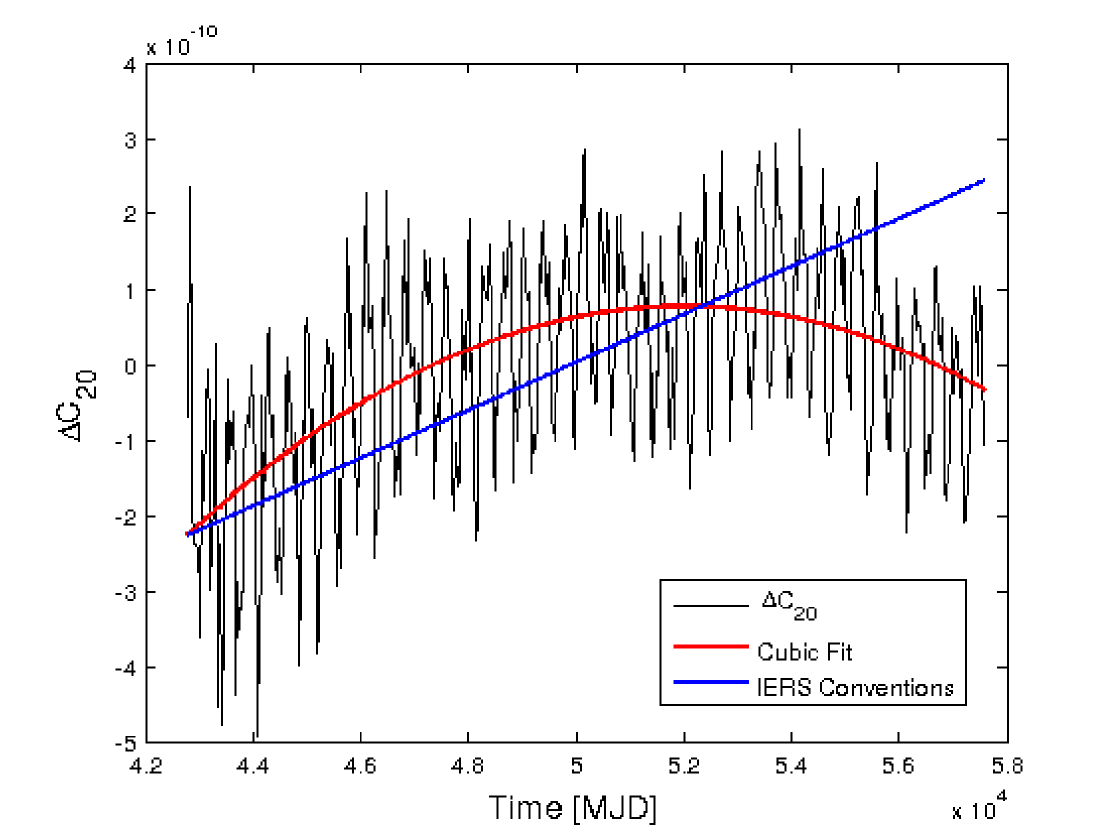


Cubic fit

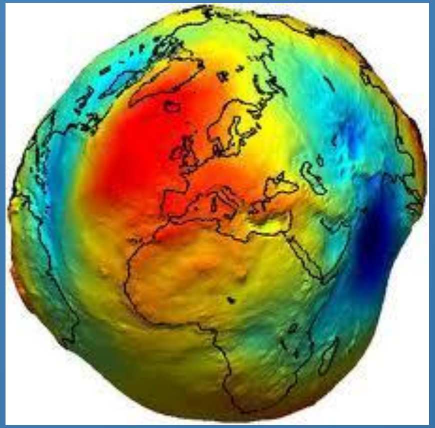
$$\Delta C_{20} = a(t_0) + b(t - t_0) + c(t - t_0)^2 + d(t - t_0)^3$$

IERS Conventions

$$\Delta C_{20} = C_{20}(t_0) + \dot{C}_{20}(t - t_0)$$

t_0 = J2000 = MJD 51544

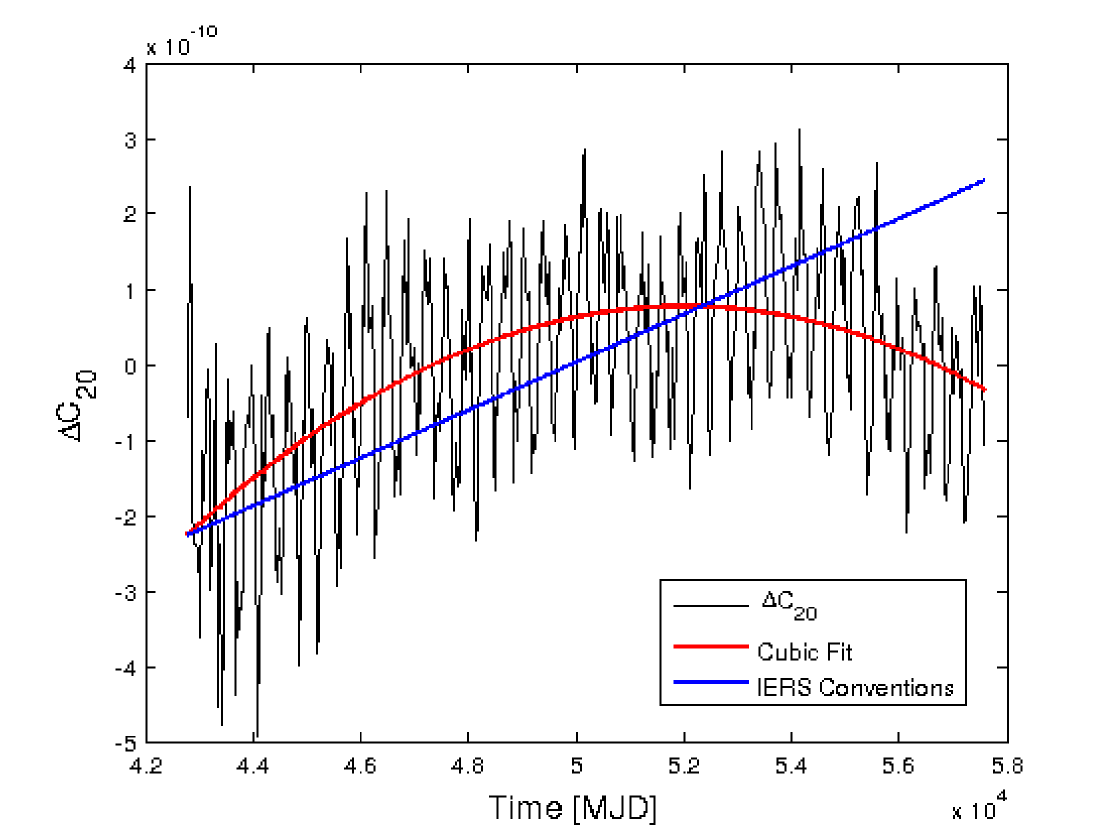
Systematics errors from the background gravitational field

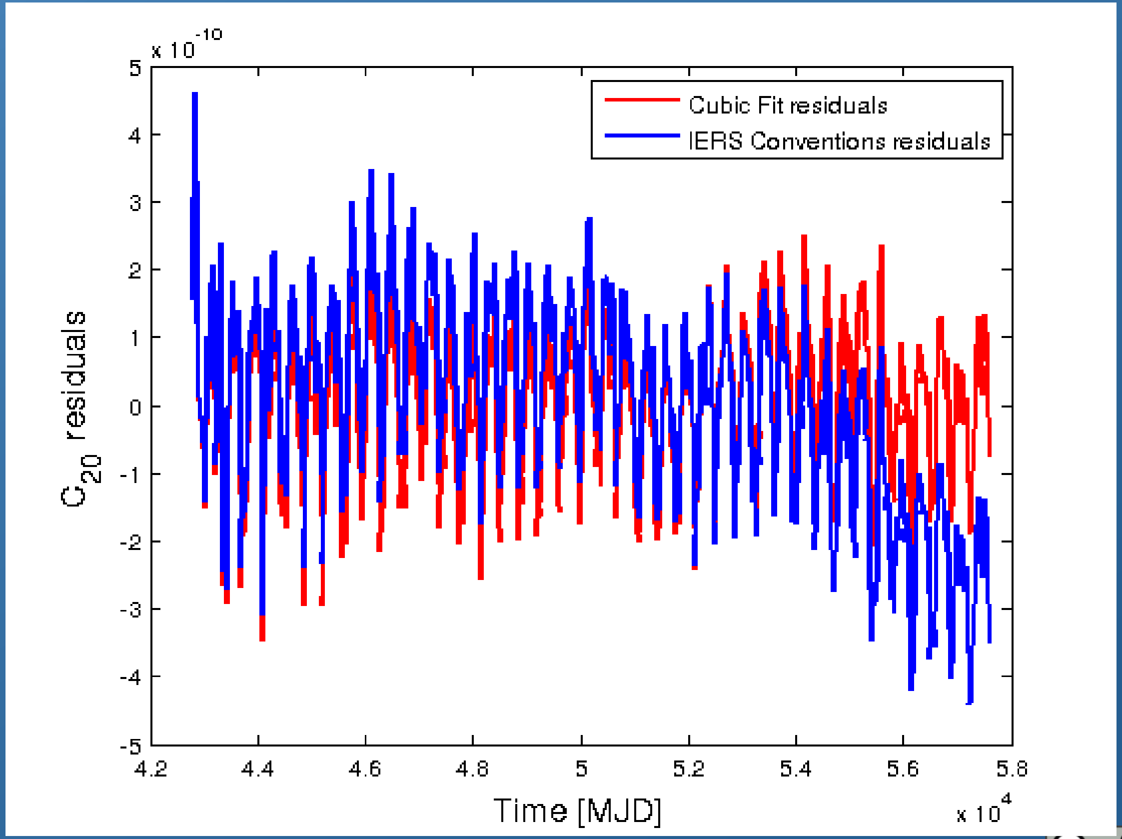


The Earth's quadrupole coefficient C20: UT/CSR monthly time series (SLR)

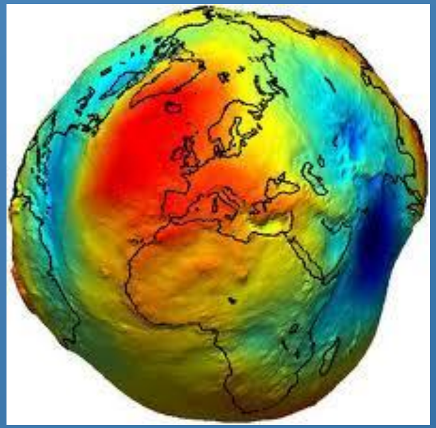
- SLR data from 8 satellites from 1976 up to 2016.5 (corrected to the tide-free system)
- $\Delta C_{20} = C_{20} - \langle C_{20} \rangle$

Cheng et al., Deceleration in Earth's oblateness. JGR, 118 (2013)





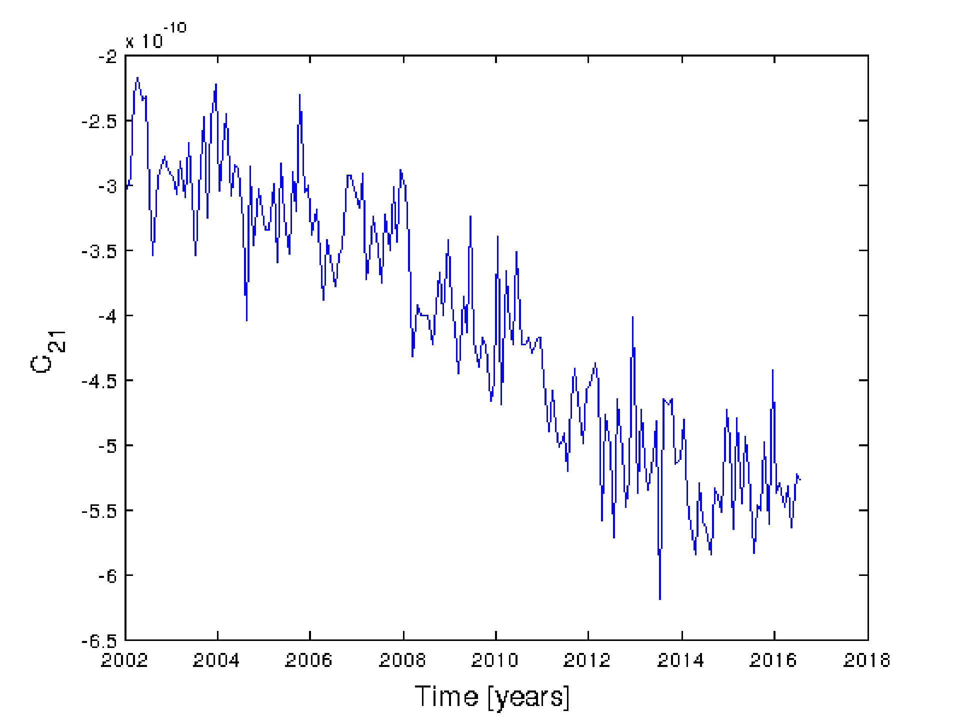
Systematics errors from the background gravitational field

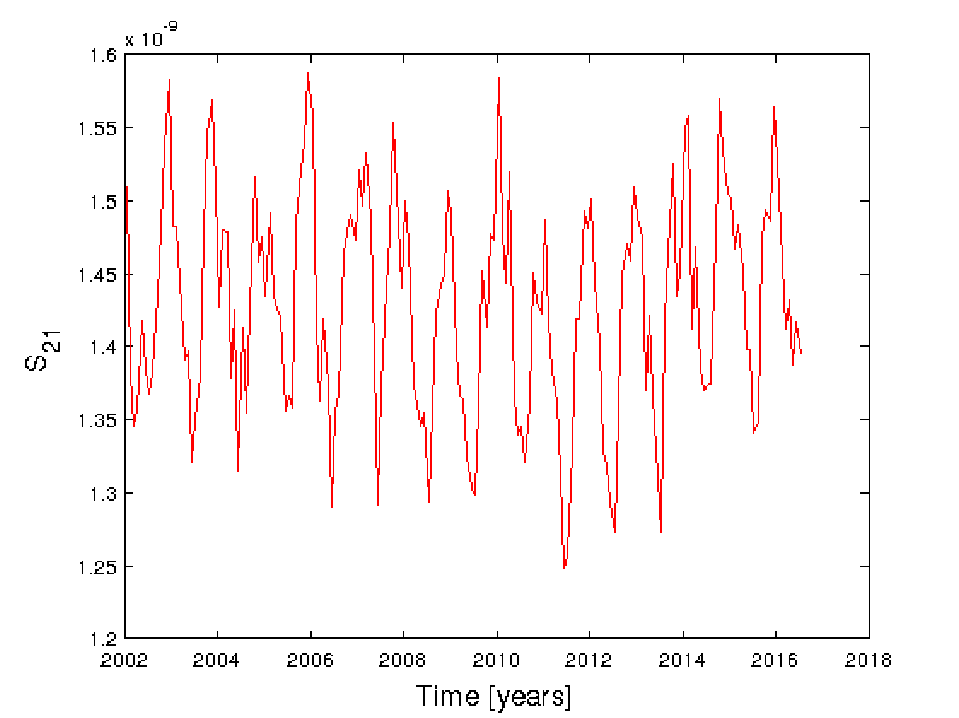


Similar considerations are valid for other low degree coefficients, as:

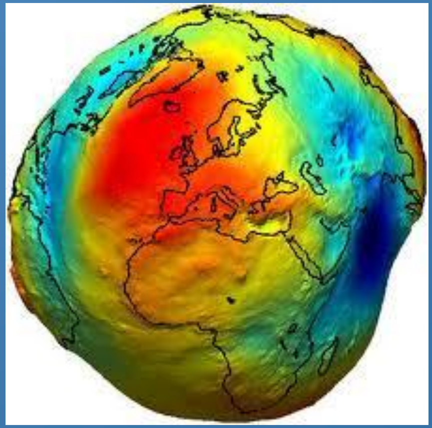
- C_{30}, C_{50}, …
- C_{40}, C_{60}, …

As well as for the coefficients that define the *figure axis* of the Earth: UT/CSR monthly time series (SLR)





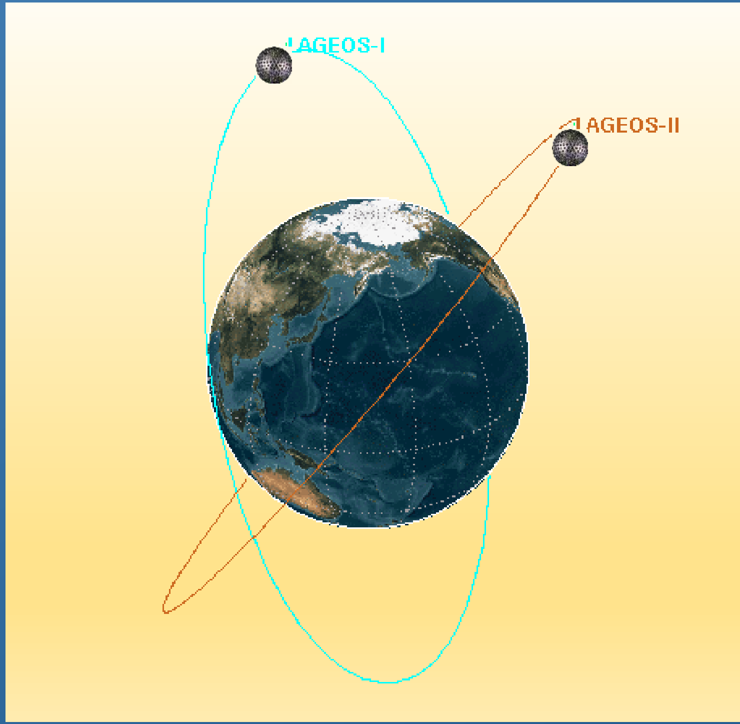
Systematics errors from the background gravitational field



Big problem with the **even zonal** harmonics uncertainties: systematic errors

We have two main unknowns:

1. the precession on the node due to the LT effect: μ_{LT} ;
2. the J_2 uncertainty: δJ_2;



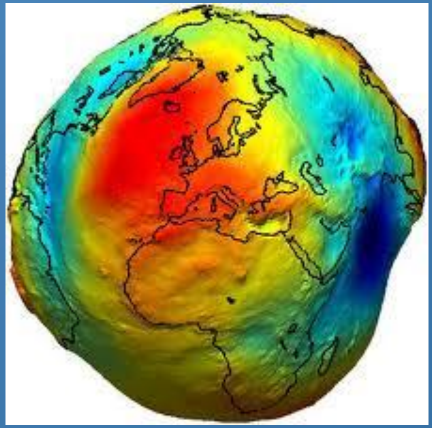
Hence, we need two observables in such a way to eliminate the uncertainty of the first even zonal harmonic and solve for the LT effect. These observables are:

1. LAGEOS node: $\delta\Omega_{Lageos}$;
2. LAGEOS II node: $\delta\Omega_{LageosII}$;

$\mu = \delta\dot{\Omega}_{I}^{res} + k\delta\dot{\Omega}_{II}^{res}$ represents the solution of a system of two equations in two unknowns

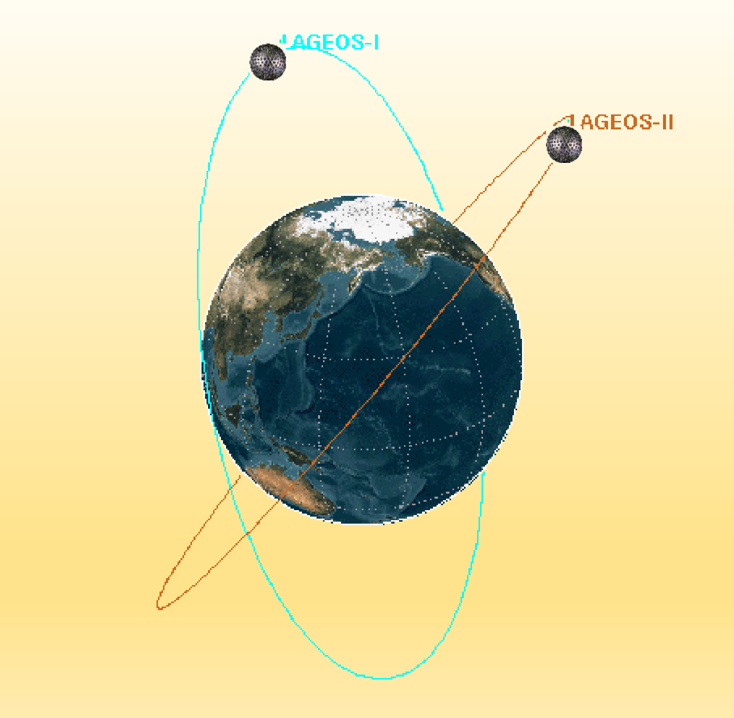
Systematics errors from the background gravitational field

Big problem with the **even zonal** harmonics uncertainties: systematic errors



With three observables we can eliminate the uncertainty of the first and second even zonal harmonics and solve for the LT effect. These observables are:

1. LAGEOS node: $\delta\Omega_{Lageos}$;
2. LAGEOS II node: $\delta\Omega_{LageosII}$;
3. LAGEOS II pericenter/LARES node $\delta\omega_{LageosII}$ / $\delta\Omega_{Lares}$;



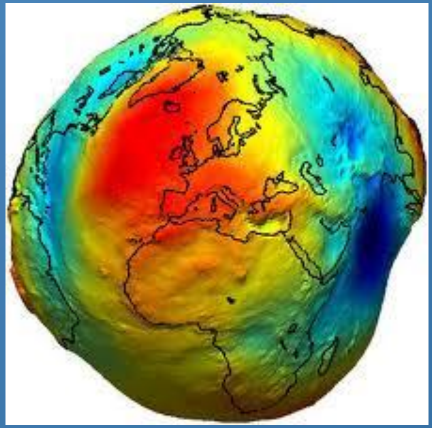
$$\mu = \delta\dot{\Omega}_I^{res} + k\delta\dot{\Omega}_{II}^{res}$$

$$\mu = \delta\dot{\Omega}_I^{res} + h_1\delta\dot{\Omega}_{II}^{res} + h_2\delta\dot{\omega}_{II}^{res}$$

$$\mu = \delta\dot{\Omega}_I^{res} + k_1\delta\dot{\Omega}_{II}^{res} + k_2\delta\dot{\Omega}_{LR}^{res}$$

Of course, including the pericenter, we have three observables: LAGEOS II perigee has been considered thanks to its larger eccentricity ($\cong 0.014$) with respect to that of LAGEOS ($\cong 0.004$) and LARES ($\cong 0.001$)

Systematics errors from the background gravitational field



Lagrange's perturbing equations and Kaula's approach:

$$V(r,\varphi,\lambda) = -\frac{GM_{\oplus}}{r}\left[1+\sum_{\ell=2}^{\infty}\sum_{m=0}^{\ell}\left(\frac{R_{\oplus}}{r}\right)^{\ell} P_{\ell m}(\sin\varphi)\left(C_{\ell m}\cos m\lambda + S_{\ell m}\sin m\lambda\right)\right]$$

$$\mathcal{R} = -V = -GM_{\oplus}\sum_{\ell,m} R_{\oplus}^{\ell} V_{\ell,m}(a,e,i,\Omega,\omega,M)$$

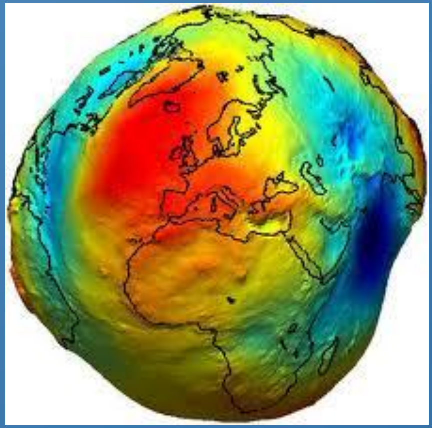
Disturbing function

$$V_{\ell,m} = \frac{1}{a^{\ell+1}}\sum_{p=0}^{p=\ell} F_{\ell,m,p}(i)\sum_{q=-\infty}^{q=+\infty} G_{\ell,m,q}(e)\left[\begin{pmatrix} C_{\ell,m} \\ -S_{\ell,m}\end{pmatrix}_{\ell-m\ odd}^{\ell-m\ even}\cos(\Psi_{\ell,m,p,q}) + \begin{pmatrix} S_{\ell,m} \\ C_{\ell,m}\end{pmatrix}_{\ell-m\ odd}^{\ell-m\ even}\sin(\Psi_{\ell,m,p,q})\right]$$

$$\Psi_{\ell,m,p,q} = (\ell-2p)\omega + (\ell-2p+q)M + m(\Omega-\theta)$$

where $F_{\ell mpq}(i)$ and $G_{\ell mq}(e)$ are the inclination and eccentricity functions of Kaula

Systematics errors from the background gravitational field



Lagrange's equations for the node and pericenter:

$$\mathcal{R} = -GM_{\oplus} \sum_{\ell,m} R_{\oplus}^{\ell} V_{\ell,m}(a, e, i, \Omega, \omega, M)$$

$$\frac{d\Omega}{dt} = -\frac{1}{na^2\sqrt{1-e^2}\sin(i)} \frac{\partial \mathcal{R}}{\partial i}$$

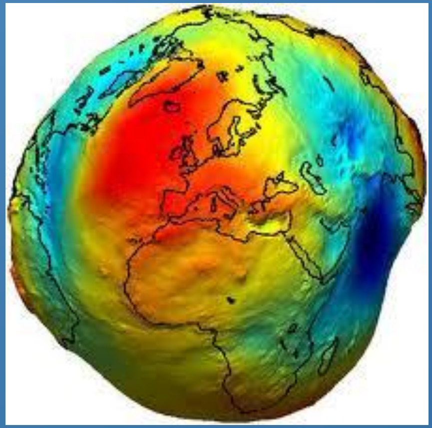
$$\frac{d\omega}{dt} = -\frac{\sqrt{1-e^2}}{na^2 e} \frac{\partial \mathcal{R}}{\partial e} + \frac{\cot(i)}{na^2\sqrt{1-e^2}} \frac{\partial \mathcal{R}}{\partial i}$$

Equations for the errors due to the even zonal harmonics (J_ℓ with ℓ=2,4,6,...):

$$\delta \frac{d\Omega}{dt} = \sum_{\ell=2}^{\ell=\ell_{max}} \left| \frac{\partial}{\partial J_\ell} \frac{d\Omega}{dt} \right|_{\ell} \delta J_\ell$$

$$\delta \frac{d\omega}{dt} = \sum_{\ell=2}^{\ell=\ell_{max}} \left| \frac{\partial}{\partial J_\ell} \frac{d\omega}{dt} \right|_{\ell} \delta J_\ell$$

Systematics errors from the background gravitational field

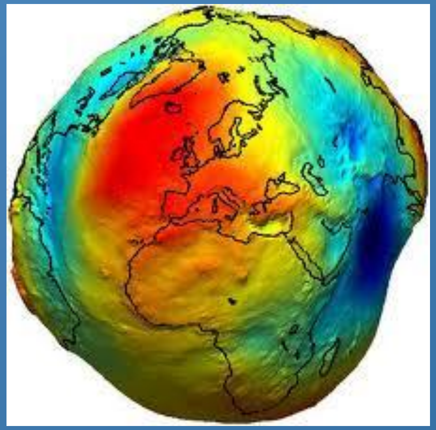


Big problem with the **even zonal** harmonics uncertainties: systematic errors

- We have considered several models for the gravitational field of the Earth in our analyses from CHAMP, GRACE and GOCE missions (and from the older multi-satellite models):

  - EIGEN2S
  - EIGEN-GRACE02S
  - GGM03S
  - GIF48
  - DGM-1S
  - GOCO03S
  - GGM05S
  - EGM2008
  - JGM-3
  - EGM96

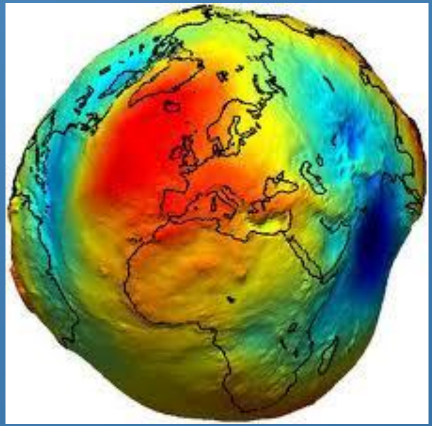
Systematics errors from the background gravitational field



Big problem with the **even zonal** harmonics uncertainties: systematic errors

- We present the results obtained for just two of these different fields:
    - GGM05S
    - EIGEN-GRACE02S
- i.e. for the fields used for the measurements performed so far for the precession of the orbital plane due to the Lense-Thirring effect

Systematics errors from the background gravitational field



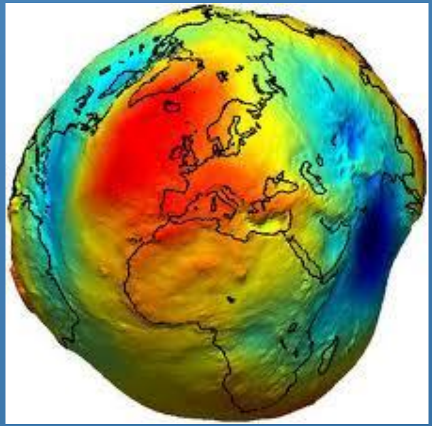
Uncertainties in the nodal rates of **LAGEOS**, **LAGEOS II** and **LARES**: **GGM05S**

$$\mu = \delta\dot{\Omega}_{I}^{res} + k_1 \delta\dot{\Omega}_{II}^{res} + k_2 \delta\dot{\Omega}_{LR}^{res}$$

Calibrated errors from GGM05S

Degree	Nodal rate errors [mas/yr] – GGM05S		
ℓ	LAGEOS	LAGEOS II	LARES
2	–109.221282927	+201.337942030	+545.564947870
4	–3.139224492	+1.140909687	+37.669353493
6	–0.950852596	–3.610259385	+25.849276956
8	–0.021732869	–0.100739170	+0.854809443
10	+0.010691329	+0.016779287	–2.344019659
12	+0.004927116	–0.007506301	–2.276900184
14	+0.001096146	–0.000009023	–1.108699479
16	+0.000133200	–0.000357766	-0.244315825
18	–0.000007894	–0.000053184	+0.146261335
20	–0.000012748	+0.000019595	+0.246592188

Systematics errors from the background gravitational field



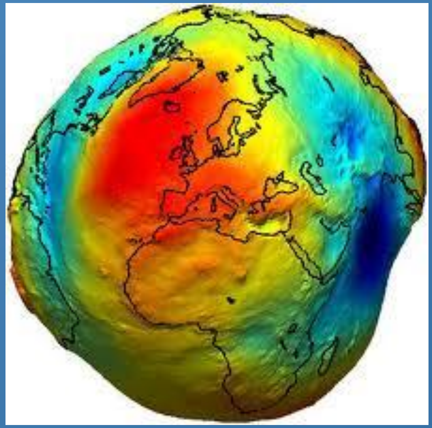
Uncertainties in the nodal rates of **LAGEOS**, **LAGEOS II** and **LARES**: **GGM05S**

$$\mu = \delta\dot{\Omega}_I^{res} + k_1\delta\dot{\Omega}_{II}^{res} + k_2\delta\dot{\Omega}_{LR}^{res}$$

Calibrated errors from GGM05S

Degree	Nodal rate errors [mas/yr] – GGM05S			Error [%]
ℓ	LAGEOS	LAGEOS II	LARES	$\delta\mu/\mu$
2	–109.221282927	+201.337942030	+545.564947870	0
4	–3.139224492	+1.140909687	+37.669353493	0
6	–0.950852596	–3.610259385	+25.849276956	0.380
8	–0.021732869	–0.100739170	+0.854809443	0.012
10	+0.010691329	+0.016779287	–2.344019659	0.308
12	+0.004927116	–0.007506301	–2.276900184	0.316
14	+0.001096146	–0.000009023	–1.108699479	0.159
16	+0.000133200	–0.000357766	-0.244315825	0.035
18	–0.000007894	–0.000053184	+0.146261335	0.021
20	–0.000012748	+0.000019595	+0.246592188	0.036

Systematics errors from the background gravitational field



Uncertainties in the nodal rates of **LAGEOS**, **LAGEOS II** and **LARES**: **GGM05S**

$$\mu = \delta\dot{\Omega}_{I}^{res} + k_1\delta\dot{\Omega}_{II}^{res} + k_2\delta\dot{\Omega}_{LR}^{res}$$

Calibrated errors from GGM05S

Degree	Nodal rate errors [mas/yr] – GGM05S			Error [%]
ℓ	LAGEOS	LAGEOS II	LARES	$\delta\mu/\mu$
2	–109.221282927	+201.337942030	+545.564947870	0
4	–3.139224492	+1.140909687	+37.669353493	0
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18	–0.000007894	–0.000053184	+0.146261335	0.021
20	–0.000012748	+0.000019595	+0.246592188	0.036

Error in the combination:

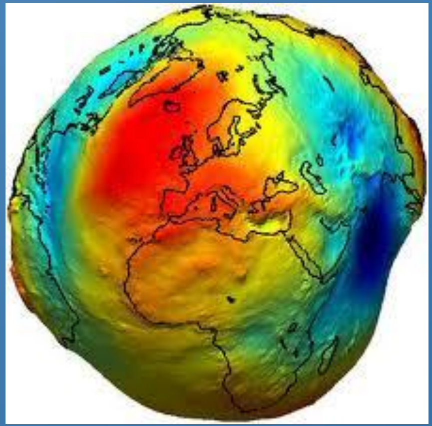
Sum Absolute Values

$$SAV = \sum_{\ell} \delta\mu_\ell \cong 1.3\%\mu$$

Root Sum Squares

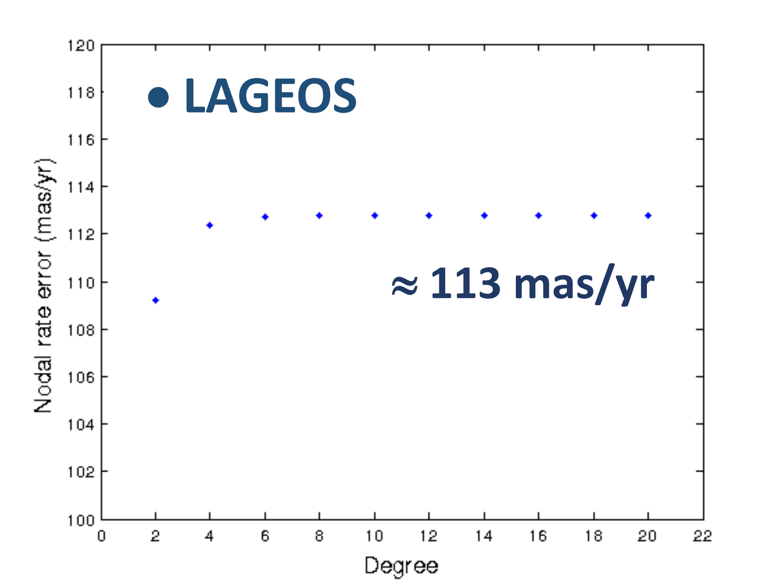
$$RSS = \sqrt{\sum_{\ell} \partial\mu_\ell^2} \cong 0.6\%\mu$$

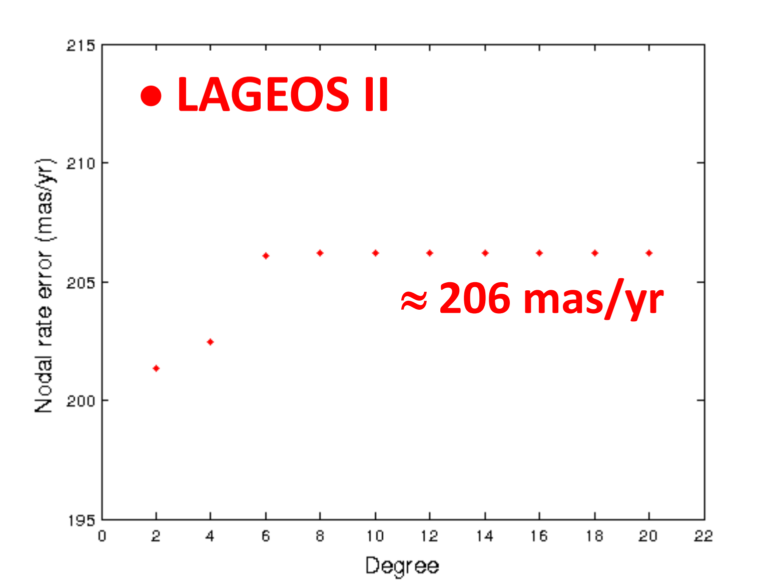
Systematics errors from the background gravitational field

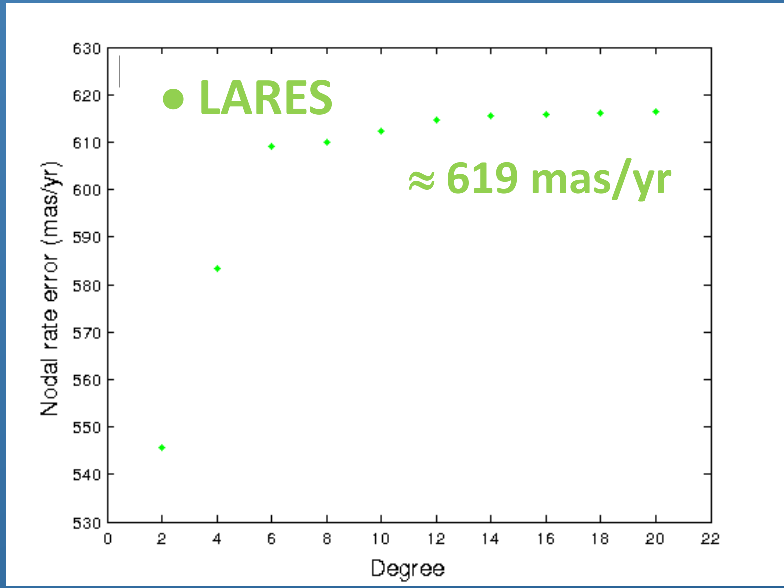


Uncertainties in the nodal rates of **LAGEOS**, **LAGEOS II** and **LARES**: **GGM05S**

Plots from previous Table

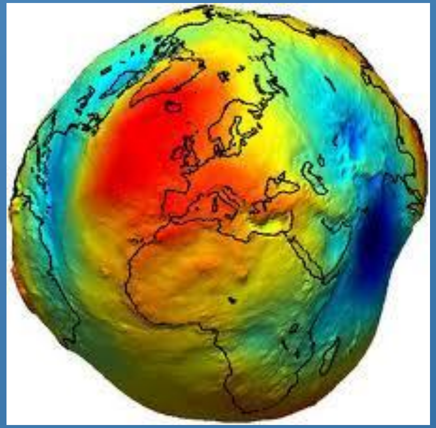






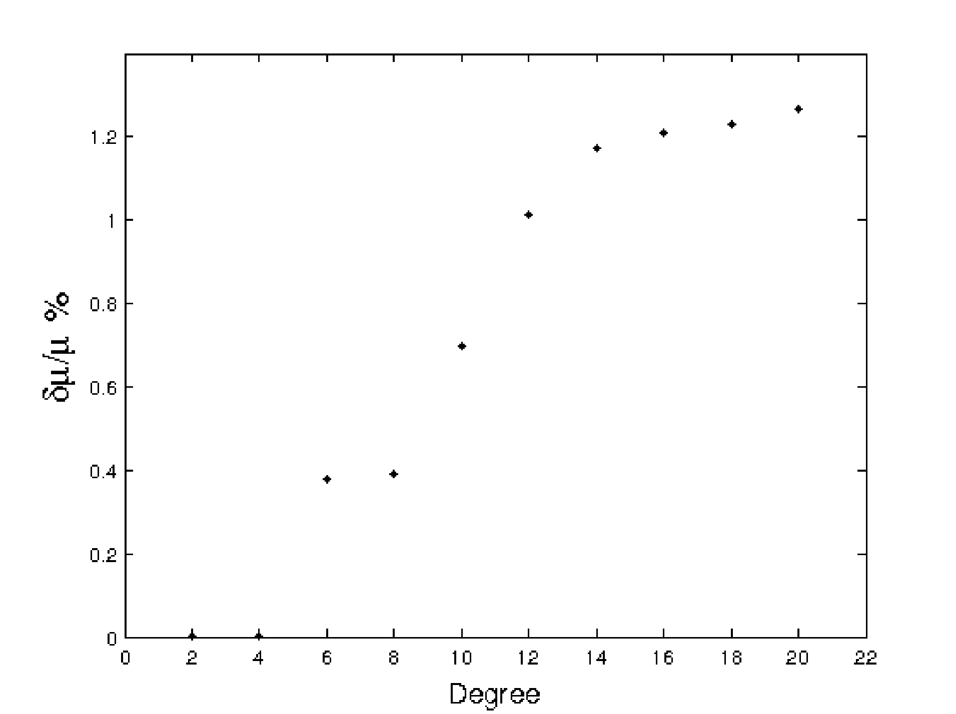
Nodal rate of the three satellites

Systematics errors from the background gravitational field



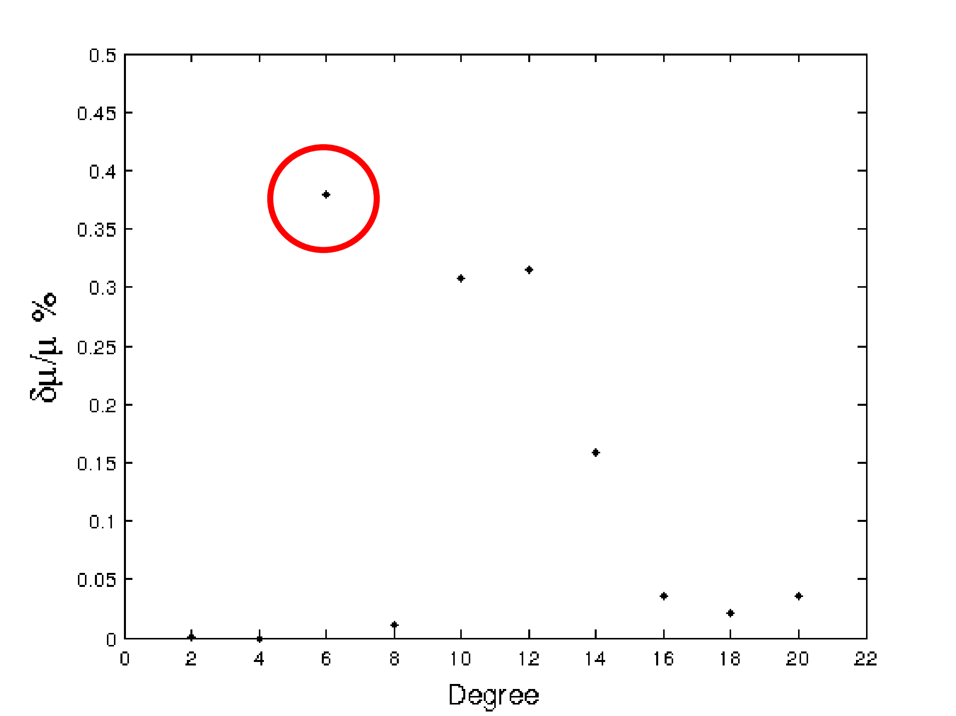
Uncertainties in the combination of the three nodal rates: **GGM05S**

Plots from previous Table



Sum Absolute Values

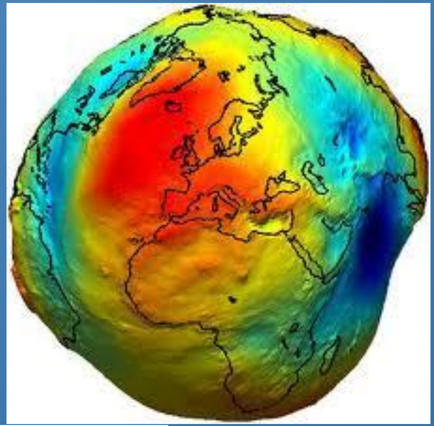
$$SAV = \sum_{\ell} \delta\mu_\ell \cong 1.3\%\mu$$



Root Sum Squares

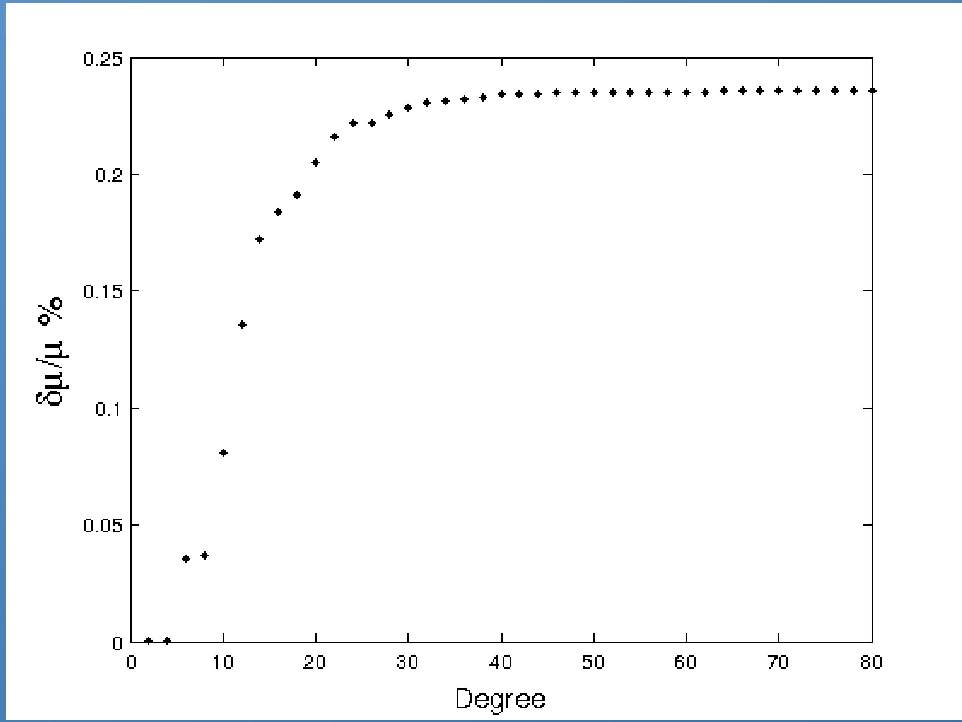
$$RSS = \sqrt{\sum_{\ell} \partial\mu_\ell^2} \cong 0.6\%\mu$$

Systematics errors from the background gravitational field



Uncertainties in the combination of the three nodal rates: **EIGEN-GRACE02S**

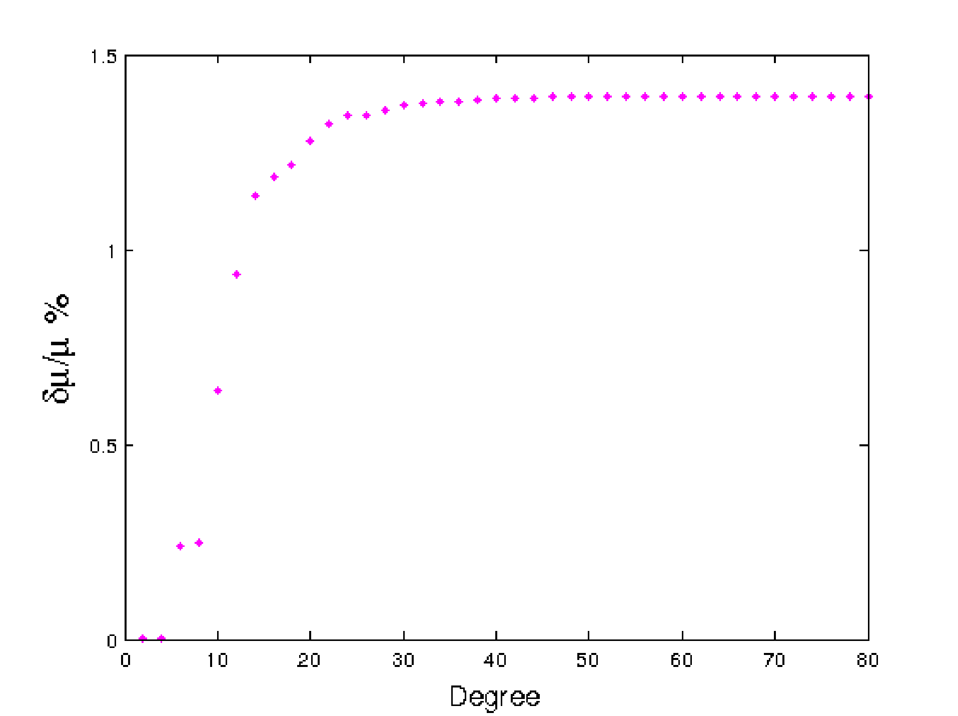
Cumulative Formal Error



Sum Absolute Values

$$SAV = \sum_{\ell} \delta\mu_{\ell} \cong 0.2\%\mu$$

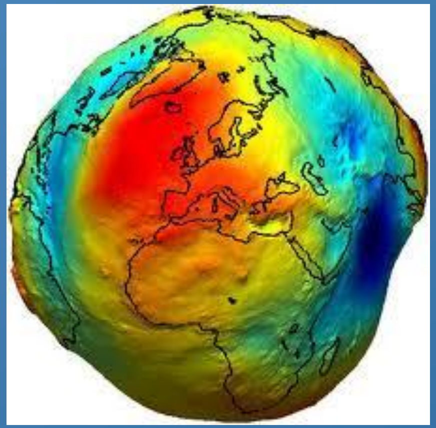
Cumulative Calibrated Error



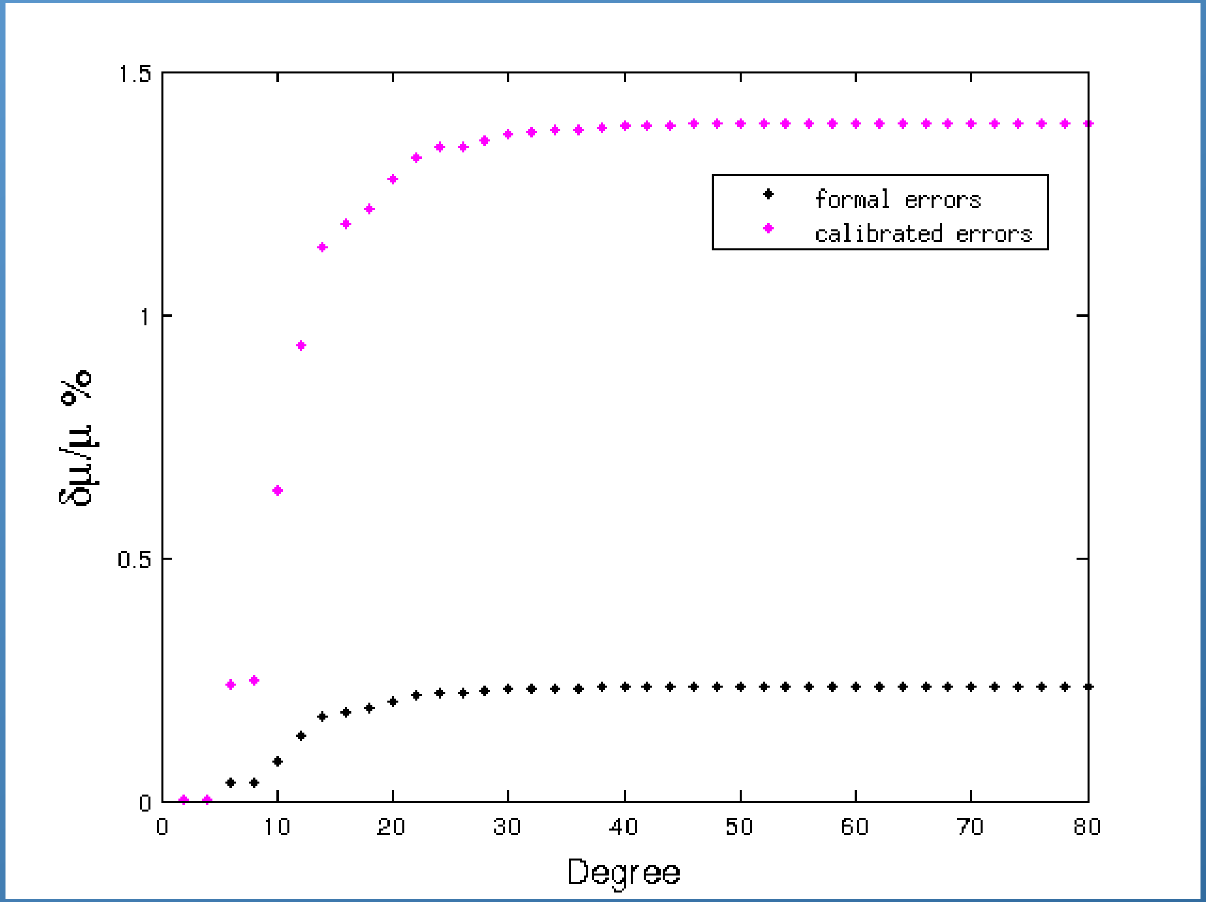
Sum Absolute Values

$$SAV = \sum_{\ell} \delta\mu_{\ell} \cong 1.4\%\mu$$

Systematics errors from the background gravitational field



Uncertainties in the combination of the three nodal rates: **EIGEN-GRACE02S**



Sum Absolute Values

$$SAV = \sum_{\ell} \delta\mu_{\ell} \cong 0.2\%\mu$$

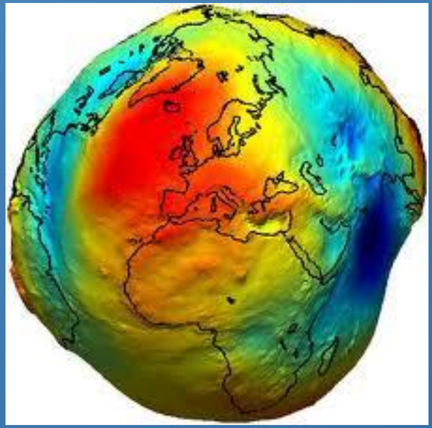
$$SAV = \sum_{\ell} \delta\mu_{\ell} \cong 1.4\%\mu$$

Root Sum Squares

$$RSS = \sqrt{\sum_{\ell} \partial\mu_{\ell}^{2}} \cong 9 \cdot 10^{-2}\%\mu$$

$$RSS = \sqrt{\sum_{\ell} \partial\mu_{\ell}^{2}} \cong 0.6\%\mu$$

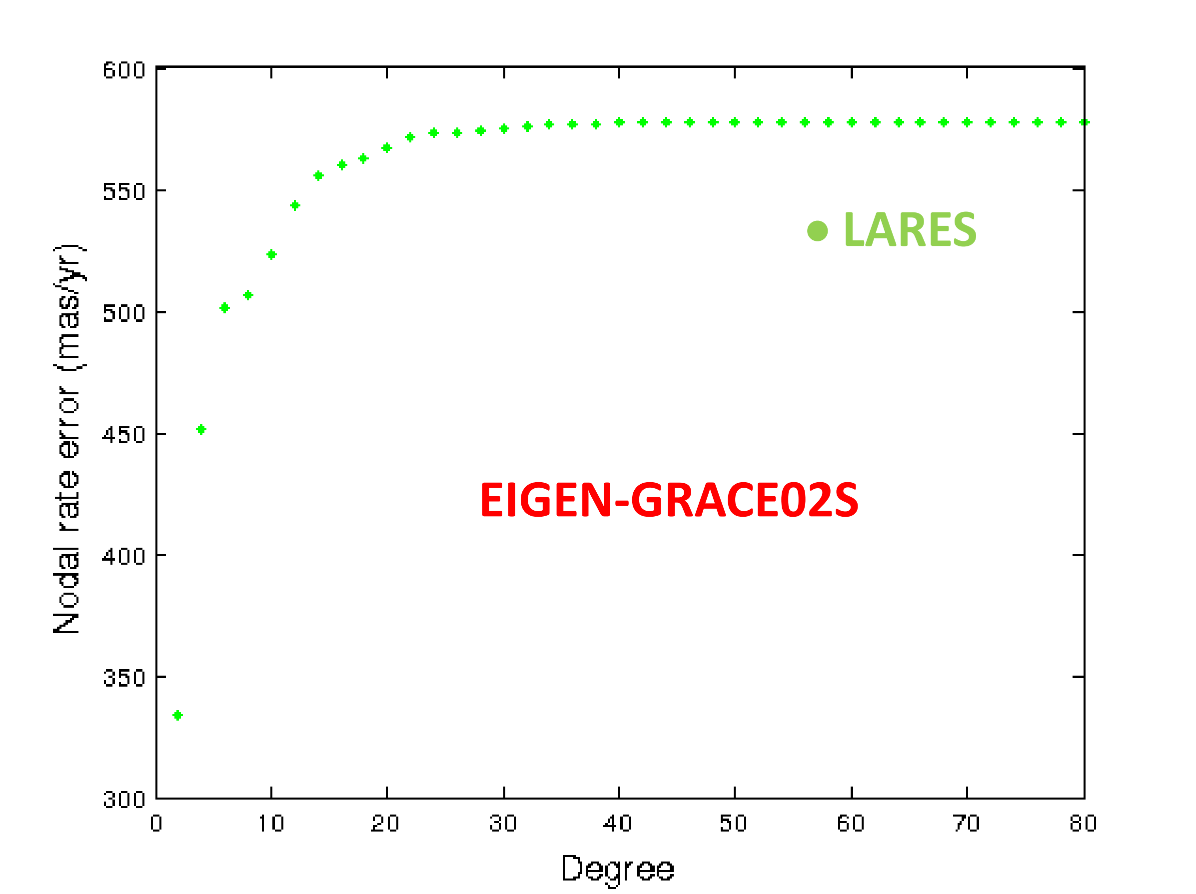
Systematics errors from the background gravitational field



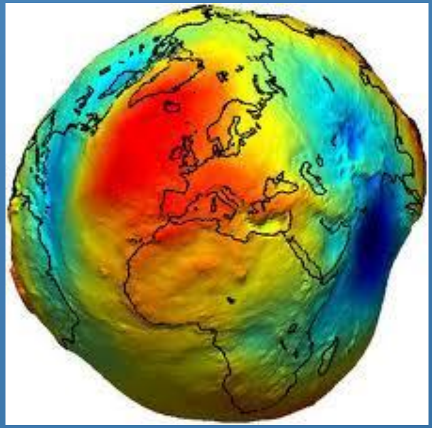
Preliminary (partial) conclusion after the comparison of different models for the Earth's gravitational field:

- in order to estimate the error budget for the Lense-Thirring effect measurement due to the uncertainties of the even zonal harmonics coefficients it is enough to consider the contribution of the harmonics up to the degree $\ell \approx 20$ or 30

- the contribution from the higher degrees is negligible, also in the case of LARES



Systematics errors from the background gravitational field



Preliminary (partial) conclusion after the comparison of different models for the Earth's gravitational field:

- we remark that, even if, in the case of LARES, the mis-modelled precession does not decrease at high degrees, we dot not report any runaway inflation of the uncertainty, as instead claimed by some authors in the literature
- possible sources of errors in the evaluation of nodal precession contribution may originate in the evaluation of the derivatives of eccentricity and inclination functions of the Kaula expansion of the potential
- to overcome computational issues of high-degree contributions, we have avoided to simplify partial sums of trigonometric terms in the form of rational expressions (with the risk of huge errors when performing derivatives)
- rather, we left expanded the trigonometric polynomials which, even if quite long, are well within the range of operation of any algebraic manipulator, resulting in accurate determination of high-degree terms

Summary

- The LARASE experiment and its goals
- Relativistic effects to be measured
- Systematic errors from the background gravitational field
- Some recent measurements of relativistic effects
- Conclusions and future work

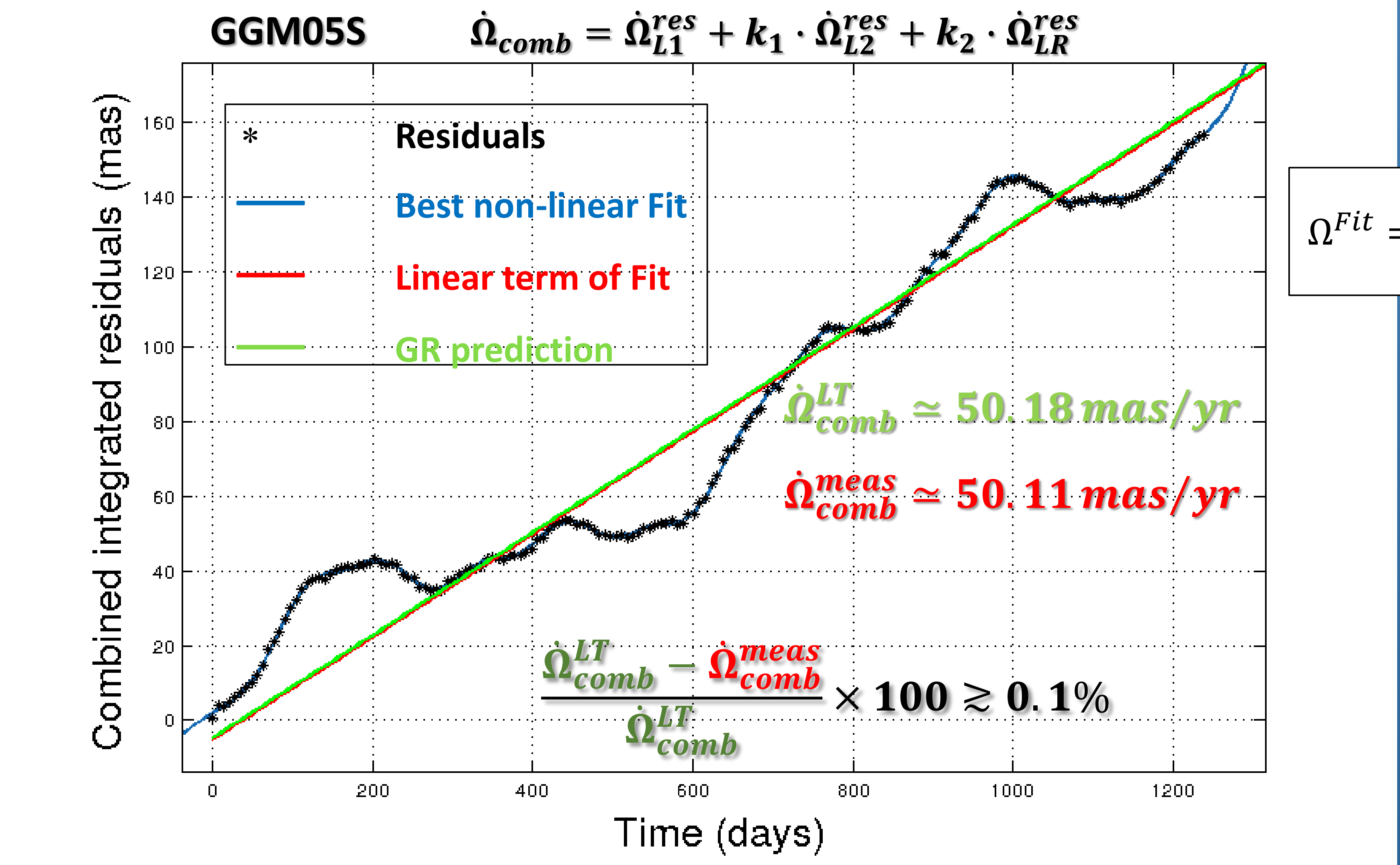
Some recent measurements of relativistic effects

LARASE measurements of relativistic precessions:

1. A new preliminary measurement of the Lense-Thirring precession with the two LAGEOS satellites and LARES (2016)
2. Measurement of the overall GR precession of LAGEOS II pericenter (2014)
3. Constraints on alternative theories of gravitation (2014)

Some recent measurements of relativistic effects

A very preliminary new measurement of the Lense-Thirring effect with the two LAGEOS and LARES (3.4 yr)



We fitted also for a minimum of three to a maximum of twelve tidal waves (both solid and ocean):

$$\Omega^{Fit} = a + b \cdot t + \sum_{i=1}^{n} A_i \cdot \sin\left(\frac{2\pi}{P_i} \cdot t + \Phi_i\right)$$

Indeed, tides mismodelling plus unmodelled nongravitational forces due to thermal effects may corrupt the measurement of the relativistic effect.

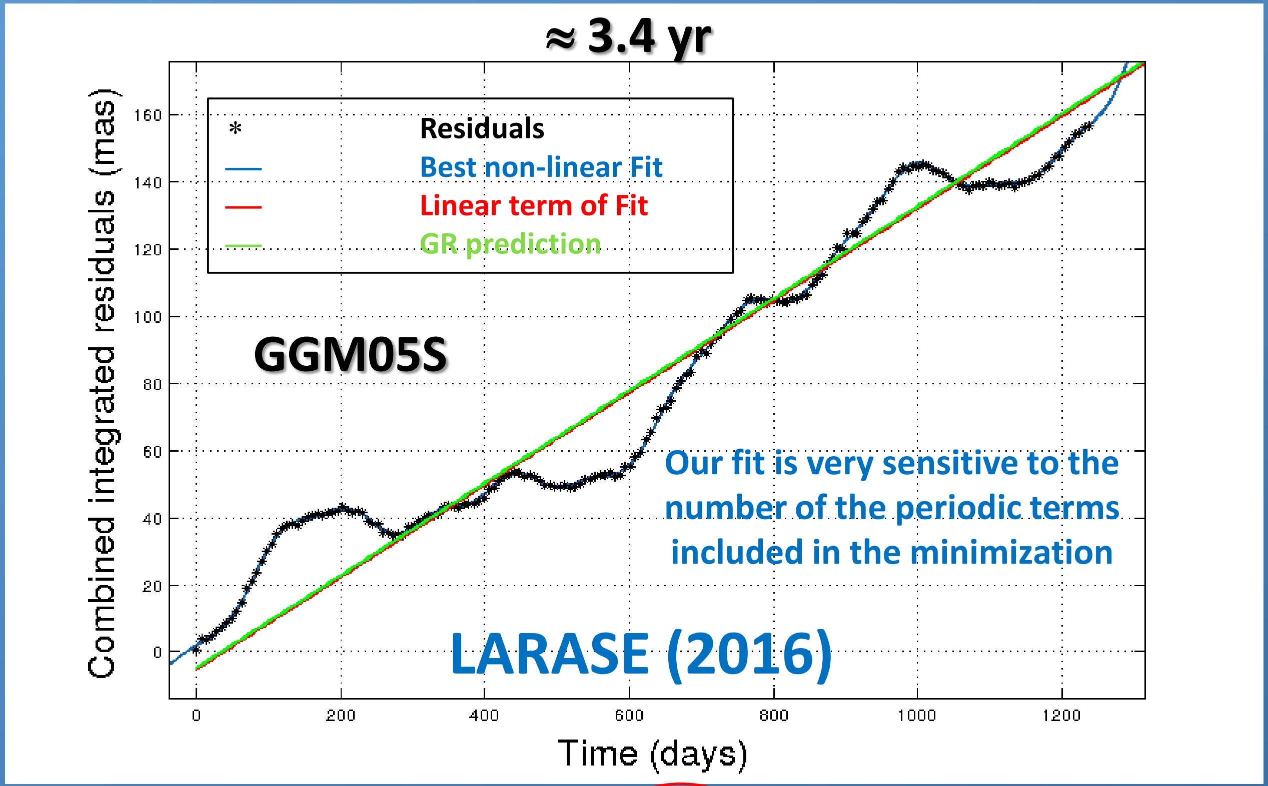
For instance, the (both solid and ocean) **K1** tides have the same periods of the right ascension of the node of the satellites:

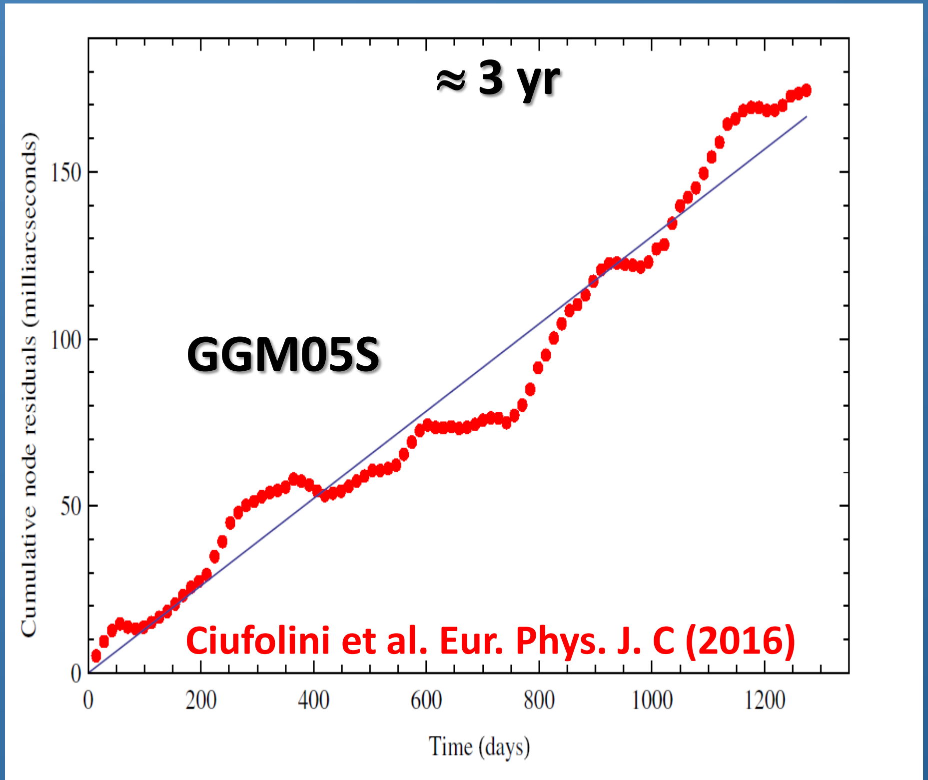
≈1044 days, ≈569 days and ≈224 days

$$\dot{\Omega}_{comb} = \dot{\Omega}^{res}_{L1} + k_1 \cdot \dot{\Omega}^{res}_{L2} + k_2 \cdot \dot{\Omega}^{res}_{LR}$$

Some recent measurements of relativistic effects

Comparison with an independent measurement





$\mu = (0.999 \pm 0.001) \pm \varepsilon(sys)$

$\mu = (0.994 \pm 0.002) \pm 0.05$

Up to a few % from a sensitivity analysis of the main tidal waves

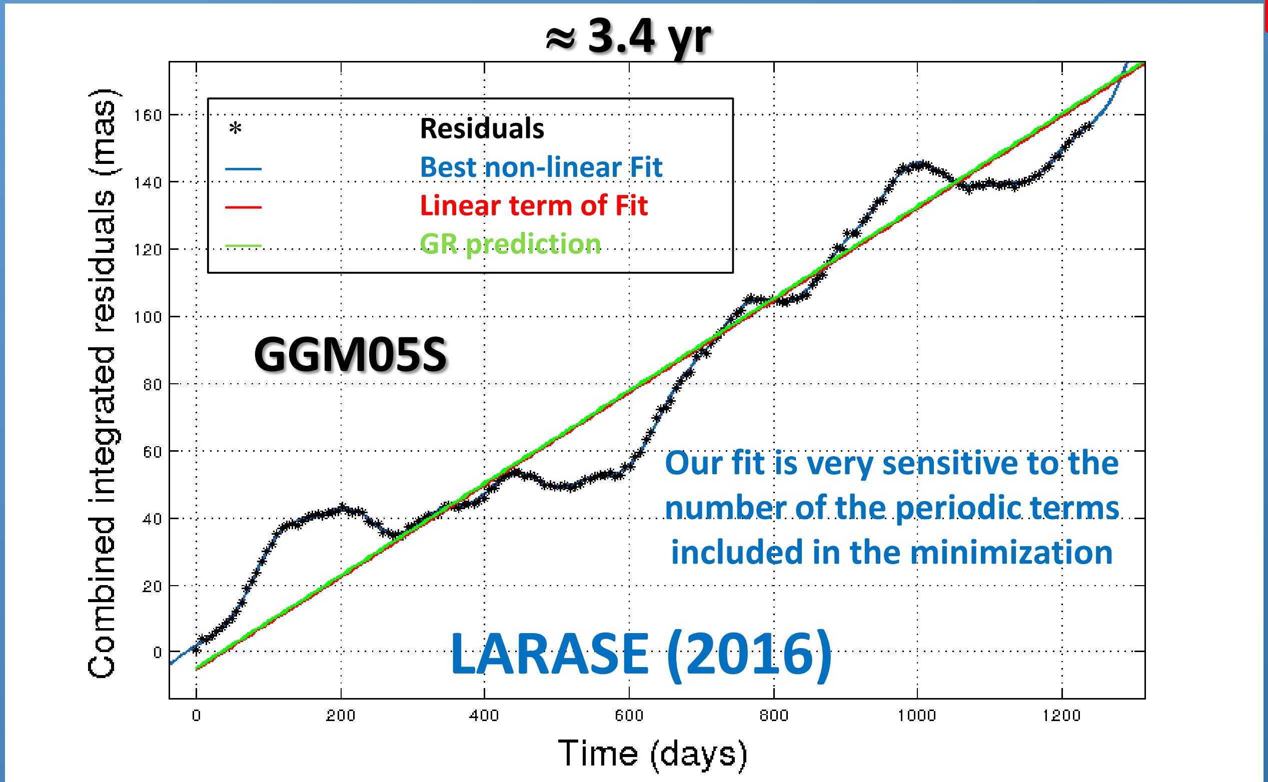
0.2% formal error of the fit (1-sigma) plus 5% preliminary estimate of systematics (4% grav. + 1% non-grav.)

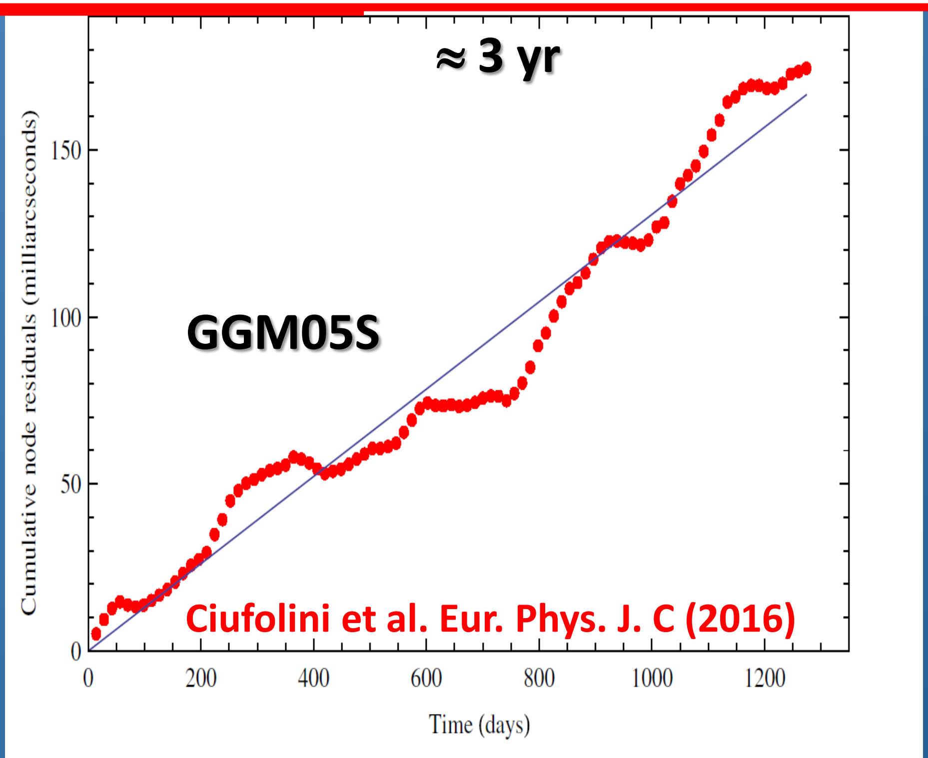
$$\dot{\Omega}_{comb} = \dot{\Omega}^{res}_{L1} + k_1 \cdot \dot{\Omega}^{res}_{L2} + k_2 \cdot \dot{\Omega}^{res}_{LR}$$

Some recent measurements of relativistic effects

Comparison with an independent measurement

Indeed, a robust and reliable estimate of systematics is one of the primary goals of LARASE !





$$\mu = (0.999 \pm 0.001\,) \pm \varepsilon(sys)$$

$$\mu = (0.994 \pm 0.002\,) \pm 0.05$$

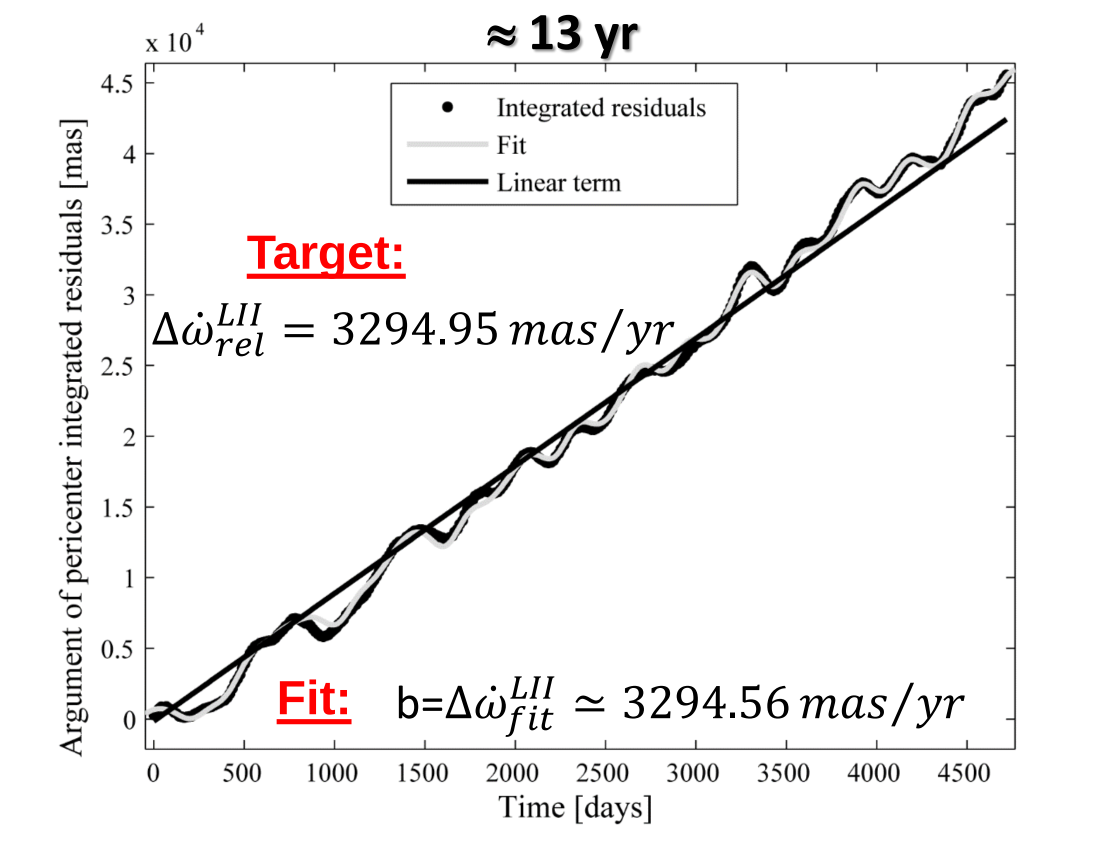
Up to a few % from a sensitivity analysis of the main tidal waves

0.2% formal error of the fit (1-sigma) plus 5% preliminary estimate of systematics (4% grav. + 1% non-grav.)

Some recent measurements of relativistic effects

A precise and accurate measurement performed in the recent pass (2014):

Fit to the pericenter residuals of LAGEOS II



Lucchesi, Peron, Phy. Rev. D, 89, 2014

Fitting function for the pericenter:

$$\Delta\omega^{FIT} = a + b\cdot t + c(t - t_0)^2 + \sum_{i=1}^{n} D_i \sin\left(\frac{2\cdot\pi}{P_i}\cdot t + \Phi_i\right)$$

- We obtained b ≅ 3294.6 mas/yr, very close to the prediction of **GR**
- The discrepancy is just **0.01%**
- From a sensitivity analysis, with constraints on some of the parameters that enter into the least squares fit, we obtained an upper bound of **0.2%**

$$\Delta\dot{\omega} = \Delta\dot{\omega}_{GP} + \Delta\dot{\omega}_{NGP} + \varepsilon\cdot\Delta\dot{\omega}_{GR}$$

$$\varepsilon = 1 - (0.12 \pm 2.10)\cdot 10^{-3} \pm 2.5\cdot 10^{-2}$$

Some recent measurements of relativistic effects

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Summary of the constraints in gravitational theories so far obtained

TABLE XVIII. Summary of the results obtained in the present work; together with the measurement error budget, the constraints on fundamental physics are listed and compared with the literature.

Parameter	Values and uncertainties (this study)	Uncertainties (literature)	Remarks
$\epsilon_\omega - 1$	$-1.2\times10^{-4}\pm2.10\times10^{-3}\pm2.54\times10^{-2}$	$\cdots$	Error budget of the perigee precession measurement in the field of the Earth
$\frac{\lvert 2+2\gamma-\beta\rvert}{3} - 1$	$-1.2\times10^{-4}\pm2.10\times10^{-3}\pm2.54\times10^{-2}$	$\pm(1.0\times10^{-3})\pm(2\times10^{-2})$[a]	Constraint on the combination of PPN parameters
$\lvert\alpha\rvert$	$\lesssim\lvert0.5\pm8.0\pm101\rvert\times10^{-12}$	$\pm1\times10^{-8}$[b]	Constraint on a possible (Yukawa-like) NLRI
$\mathcal{C}_{\oplus\mathrm{LAGEOSII}}$	$\leq(0.003\ \mathrm{km})^4\pm(0.036\ \mathrm{km})^4\pm(0.092\ \mathrm{km})^4$	$\pm(0.16\ \mathrm{km})^4$[c]; $\pm(0.087\ \mathrm{km})^4$[d]	Constraint on a possible NSGT
$\lvert 2t_2+t_3\rvert$	$\lesssim3.5\times10^{-4}\pm6.2\times10^{-3}\pm7.49\times10^{-2}$	3×10^{-3}[e]	Constraint on torsion

[a] From the preliminary estimate of the systematic errors of [166] for the perihelion precession of Mercury.
[b] From [167] with Lunar-LAGEOS *GM* measurements.
[c] From [5] and based on a partial estimate for the systematic errors.
[d] From [7] and based on the analysis of the systematic errors only.
[e] From [168] with no estimate for the systematic errors.

Some recent measurements of relativistic effects

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$\lvert\alpha\rvert$	$\lesssim \lvert 0.5 \pm 8.0 \pm 101 \rvert \times 10^{-12}$	$\pm 1 \times 10^{-8b}$	Constraint on a possible (Yukawa-like) NLRI
$C_{\oplus \mathrm{LAGEOSII}}$	$\leq (0.003\ \mathrm{km})^4 \pm (0.036\ \mathrm{km})^4 \pm (0.092\ \mathrm{km})^4$	$\pm(0.16\ \mathrm{km})^{4c}$; $\pm(0.087\ \mathrm{km})^{4d}$	Constraint on a possible NSGT
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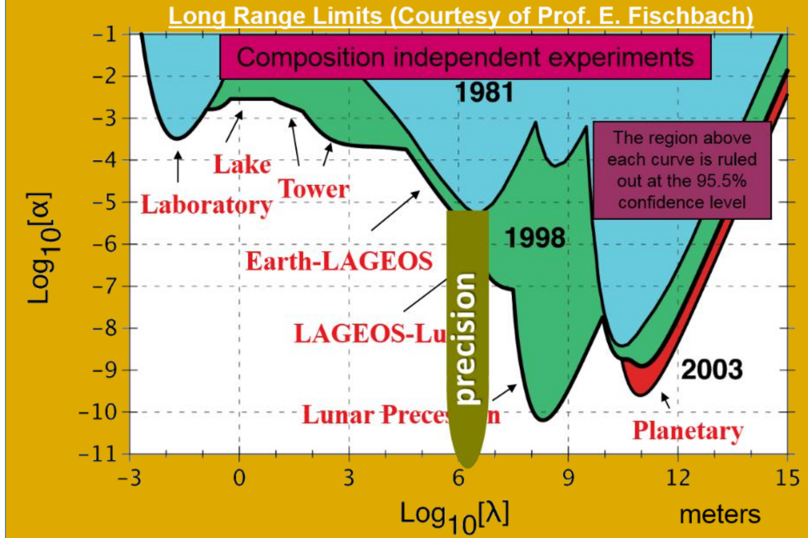
- **Constraints on a long–range force: Yukawa–like interaction**



Reference: Coy, Fischbach, Hellings, Standish, & Talmadge (2003)

Conclusions and future work

The **LARASE** (**LA**ser **RA**nged **S**atellites Experiment) activities, in terms of orbit modelling improvements and relativistic measurements, are ongoing. In particular:

- the preliminary measurements of relativistic effects are very promising and very precise (thanks to both models and POD improvements)
- a deep evaluation of the main systematic error sources has been started
- we presented the results for the even zonal harmonics uncertainties (calibrated and not) of various models of the Earth's gravitational field up to a very high degree for its expansion in spherical harmonics
- in order to have a reliable evaluation for the error it is enough to consider harmonics up to degree $\approx$30, at least in the case of the Lense-Thirring effect measurement
- of course, other additional analysis are also necessary to finally reach a very robust error budget

Conclusions and future work

Future work:

- a new study has been started in order to improve the thermal models of the two LAGEOS and to develop a thermal model for LARES by means of a Finite Element Model (FEM)
- thermal effects are the main candidate in order to explain a residual along-track deceleration on LARES of the order of 2×10^{-13} m/s^2 (not explained by the neutral drag), but charged drag will be also investigated
- the POD set up (stations position/velocity and biases, International Conventions/Reference frames, etc.) is in line with that of the Analysis Centers of the ILRS and is continuously improving:
  - ✓ POD is very good for the two LAGEOS
  - ✓ but some improvement is still expected for LARES
- new measurements of relativistic effects in the field of the Earth are foreseen

More on LARASE activities

@ Poster Session

Poster X3.120 EGU2017-13124

LARASE

LASERRANGEDSATELLITESEXPERIMENT

LARASE website

http://larase.roma2.infn.it

IOP Publishing Classical and Quantum Gravity

Class. Quantum Grav. 32 (2015) 155012 (50pp) doi:10.1088/0264-9381/32/15/155012

Testing the gravitational interaction in the field of the Earth via satellite laser ranging and the Laser Ranged Satellites Experiment (LARASE)

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Received 24 March 2015, revised 1 June 2015
Accepted for publication 15 June 2015
Published 14 July 2015

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Many thanks for your kind attention