Karlsruhe Institute of Technology

GIK

Geophysical Institute KIT-Dept. of Physics

Height Datum Unification with Sparse Gravity Data based on the Fixed Geodetic Boundary Value Problem

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Outline

- Fixed Geodetic Boundary Value Problem (GBVP)
- Minimizing the truncation error
- Determining height datum offsets
- Conclusion and outlook

Fixed Geodetic Boundary Value Problem

- Boundary values: gravity **disturbances**

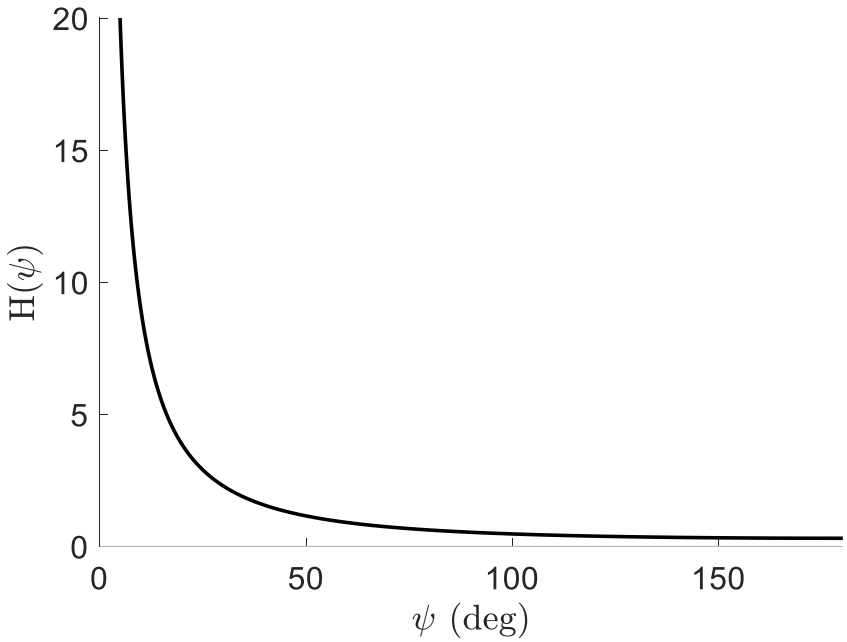
$$\delta g = g(P) - \gamma(P), \qquad P \text{ on Earth's surface}$$

  ⟹ for $\gamma(P)$, latitude and **ellipsoidal heights** must be known (GNSS)

- Linearized, spherical boundary condition: $\delta g = -\frac{\partial T}{\partial r}$

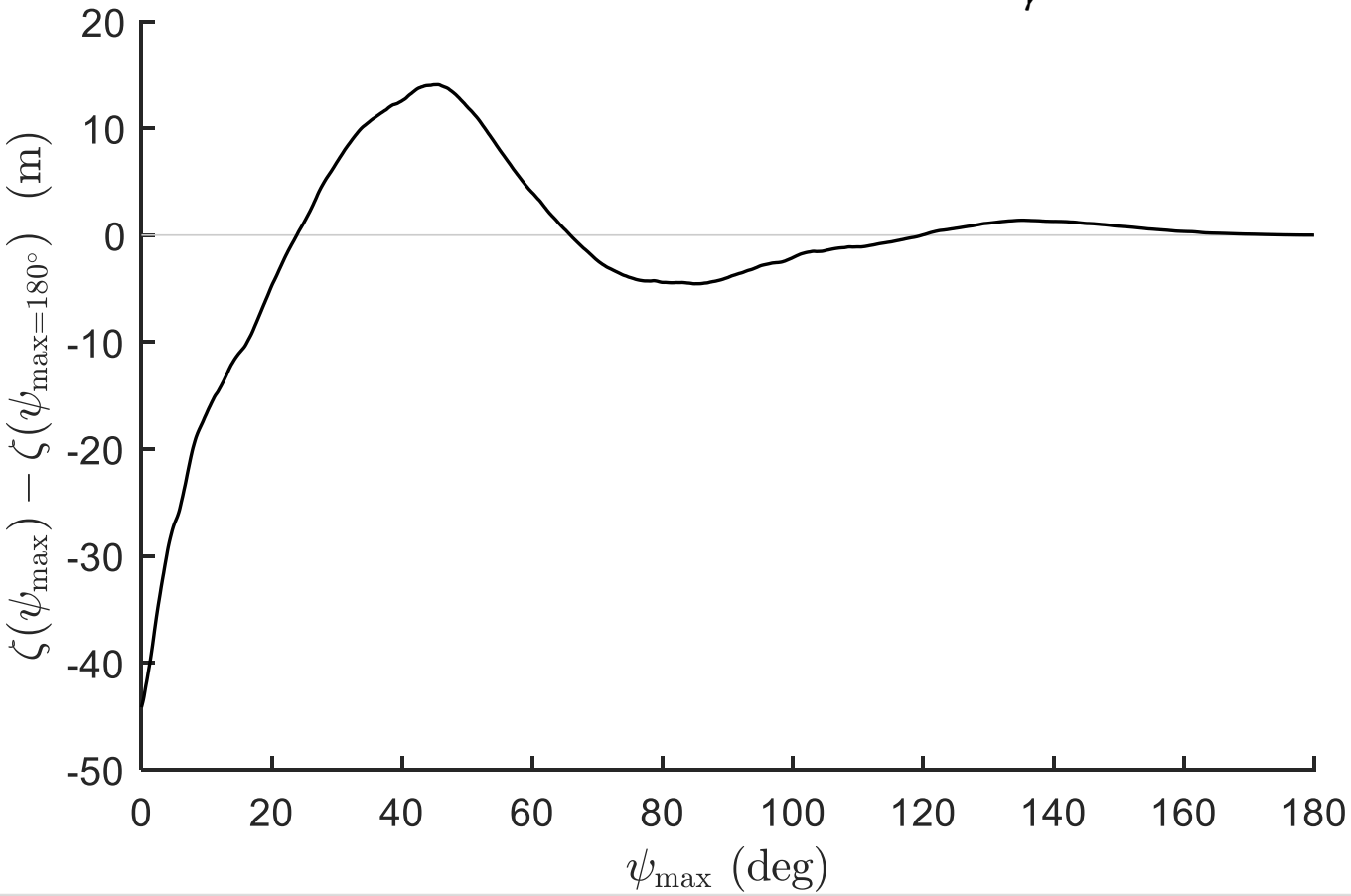
- Approx. solution:

$$T \approx \frac{R}{4\pi} \iint_{\sigma} \delta g \cdot \mathrm{H}(\psi)\, \mathrm{d}\sigma$$



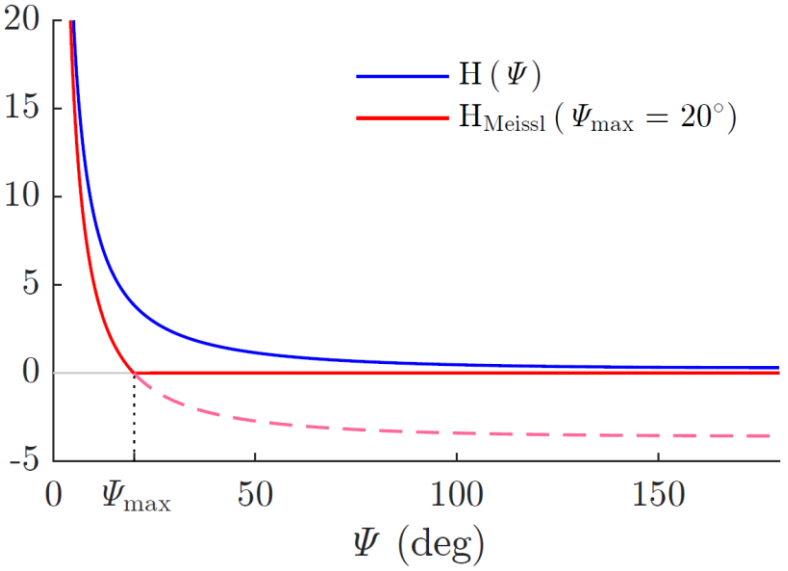
Truncation error

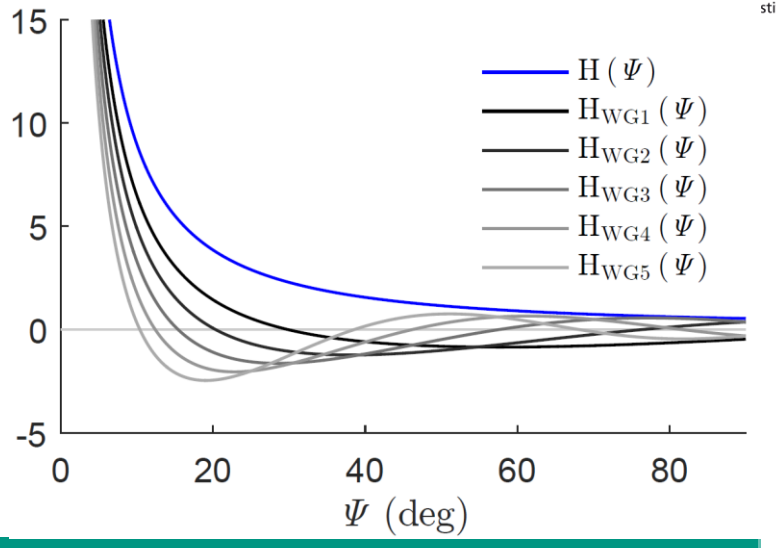
- Data: EGM2008-derived gravity disturbances (5´ x 5´) in spherical approximation on the surface of a sphere
- Test point in central Europe, height anomaly $\zeta = \frac{T}{\overline{\gamma}}$



Modifications of Hotine's function

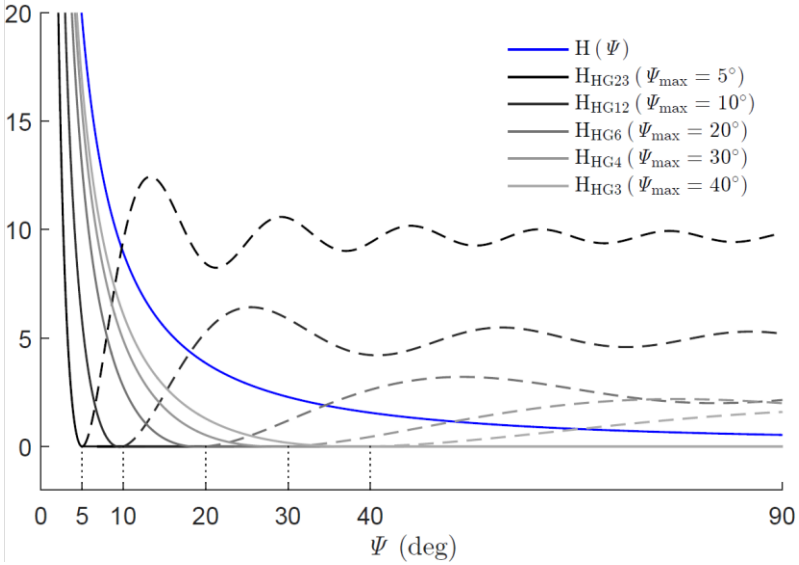
Meissl (1971)





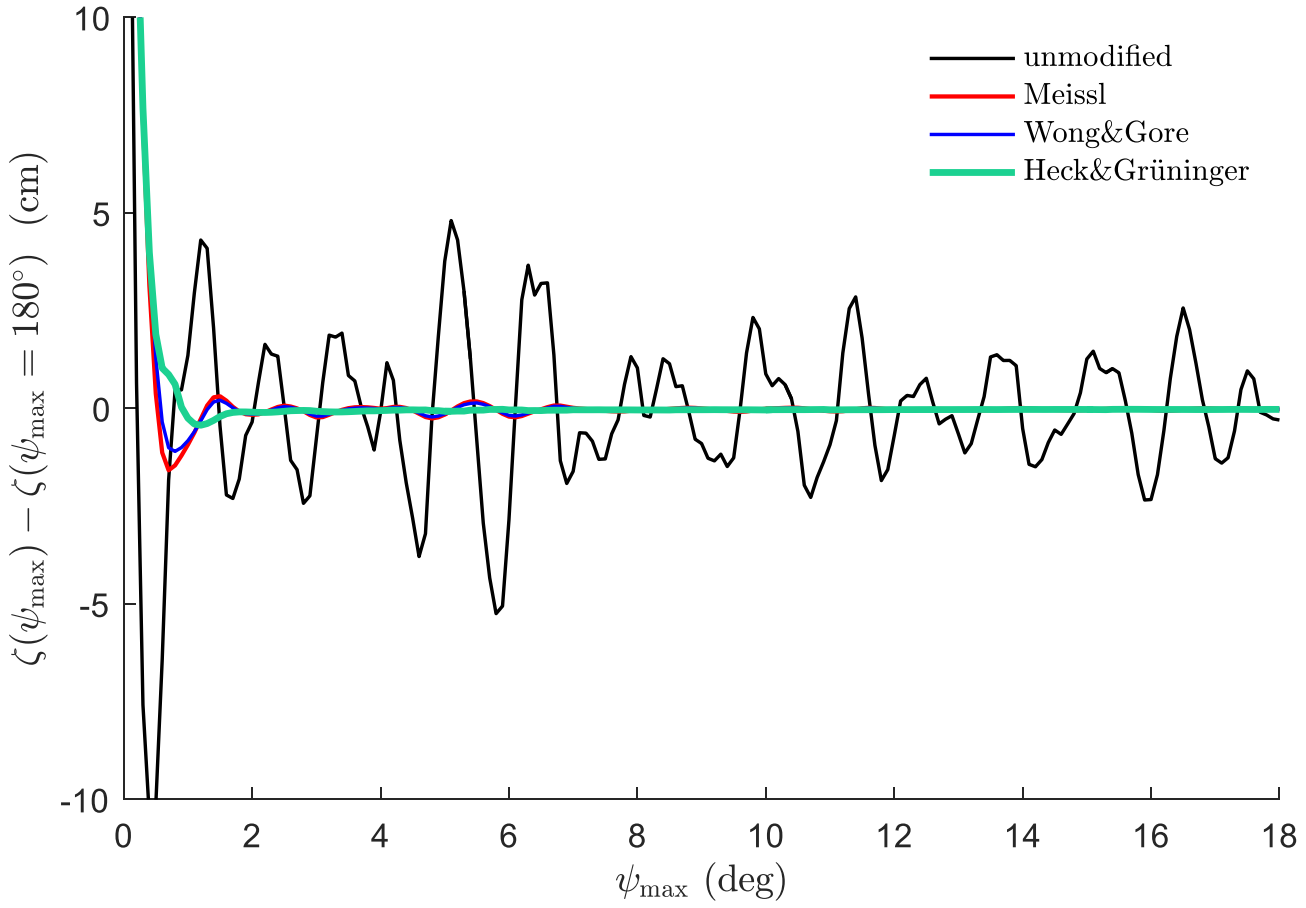
Wong & Gore (1969)

Heck & Grüninger (1987)



Incorporating a geopotential model (*N*=200)

$$\zeta = \zeta^{N}_{\mathrm{GGM}} + \frac{R}{4\pi\bar{\gamma}} \cdot \iint (\delta g - \delta g^{N}_{\mathrm{GGM}}) \cdot \mathrm{H}(\psi)\, \mathrm{d}\sigma$$



Geodetic Institute, Department of Engineering and Geoscience
Geophysical Institute, Department of Physics

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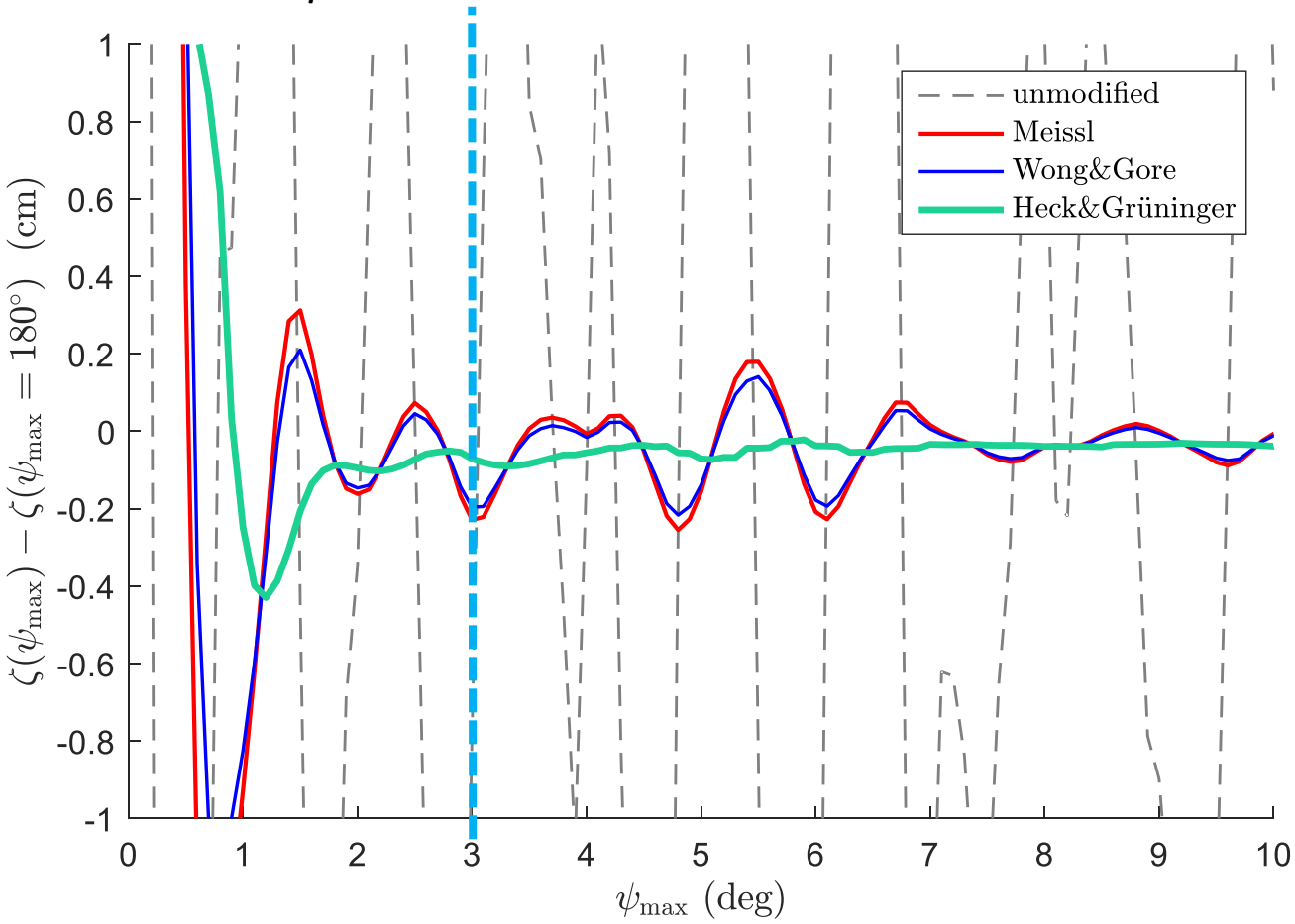
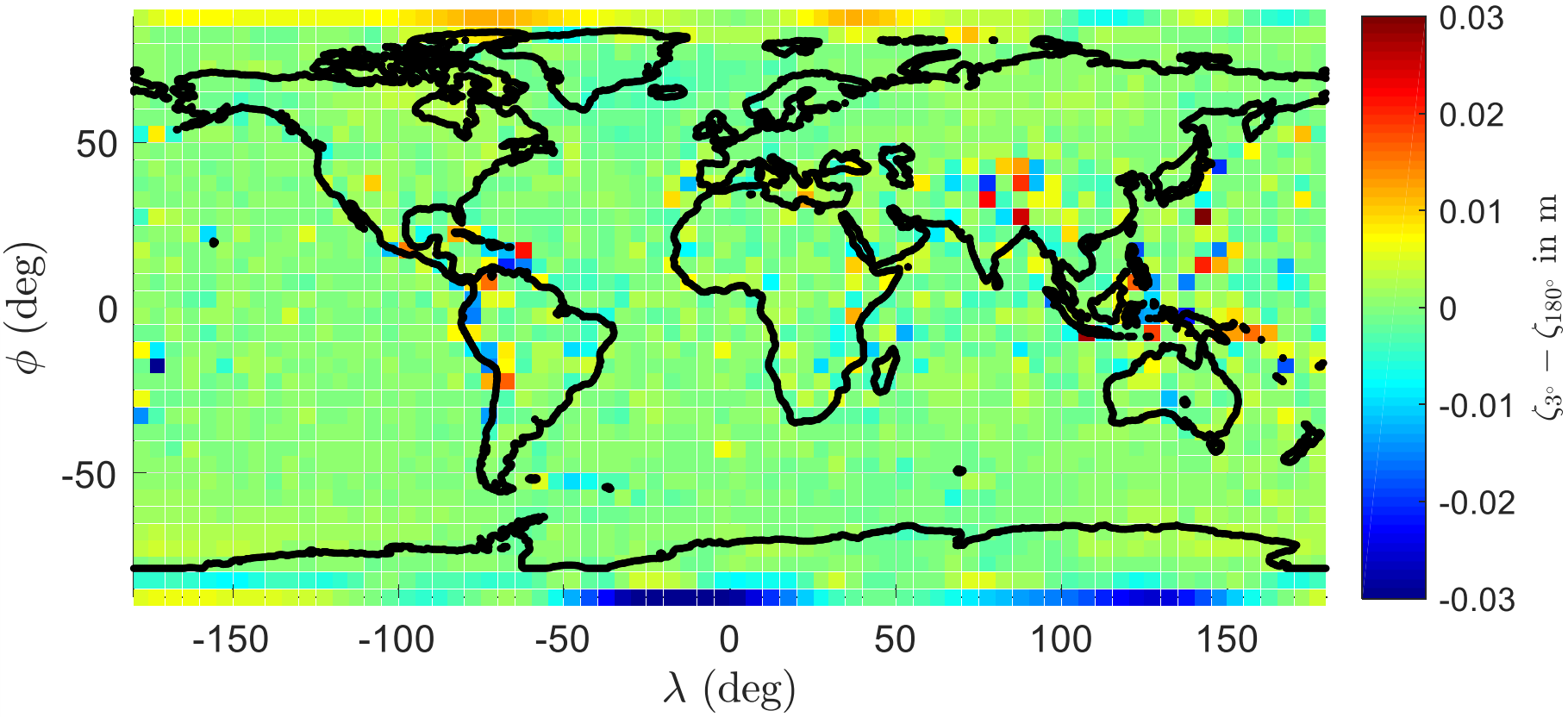


Truncation Error

- Modification acc. Heck&Grüninger ($N = 200$), $\psi_{\max} = 3^{\circ}$

RMS = 4.23 mm



Unification of height datums
Closed-Loop-Simulation for offset determination

- Transformation of gravity anomalies to disturbances
- Numerical integration at GNSS-leveling Points P_j^i

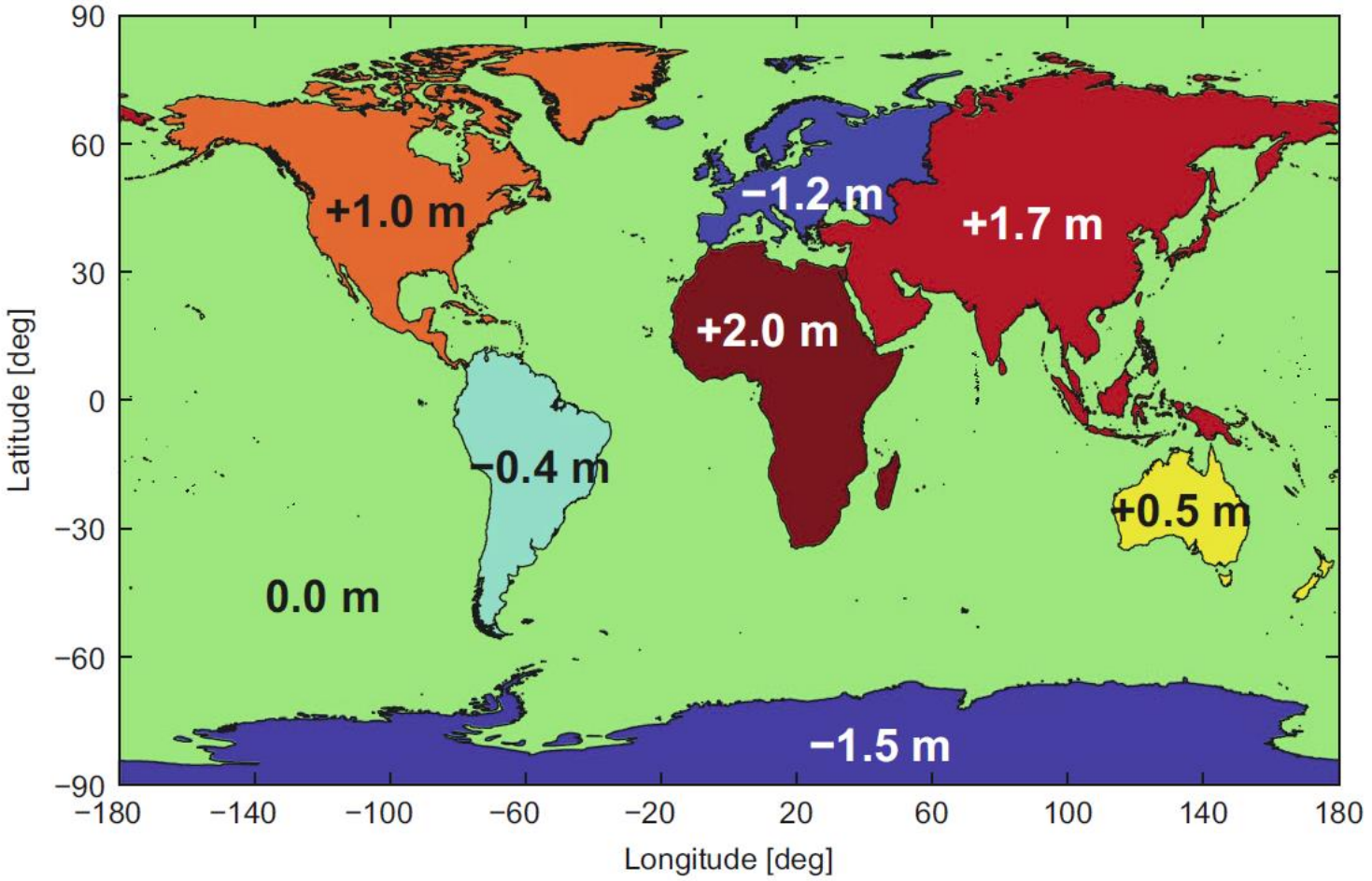
$$\overbrace{\zeta^i - \frac{R}{4\pi\gamma}\iint_{\sigma_k} \mathrm{H}(\psi)\left(\Delta g^i + \frac{2}{R}T\right)\,\mathrm{d}\sigma\Big|_{P_j^i}}^{\boldsymbol{l}} = \delta H^i + \sum_{k=1}^{n}\left\{\delta H^k \cdot \frac{1}{2\pi}\iint_{\sigma_k}\mathrm{H}(\psi)\,\mathrm{d}\sigma\Big|_{P_j^i}\right\}$$

- Estimation of height datum offsets δH^i:

$$\delta\widehat{H} = (\mathbf{A}^\mathbf{T}\mathbf{A})^{-1}\mathbf{A}^\mathbf{T}\boldsymbol{l}$$

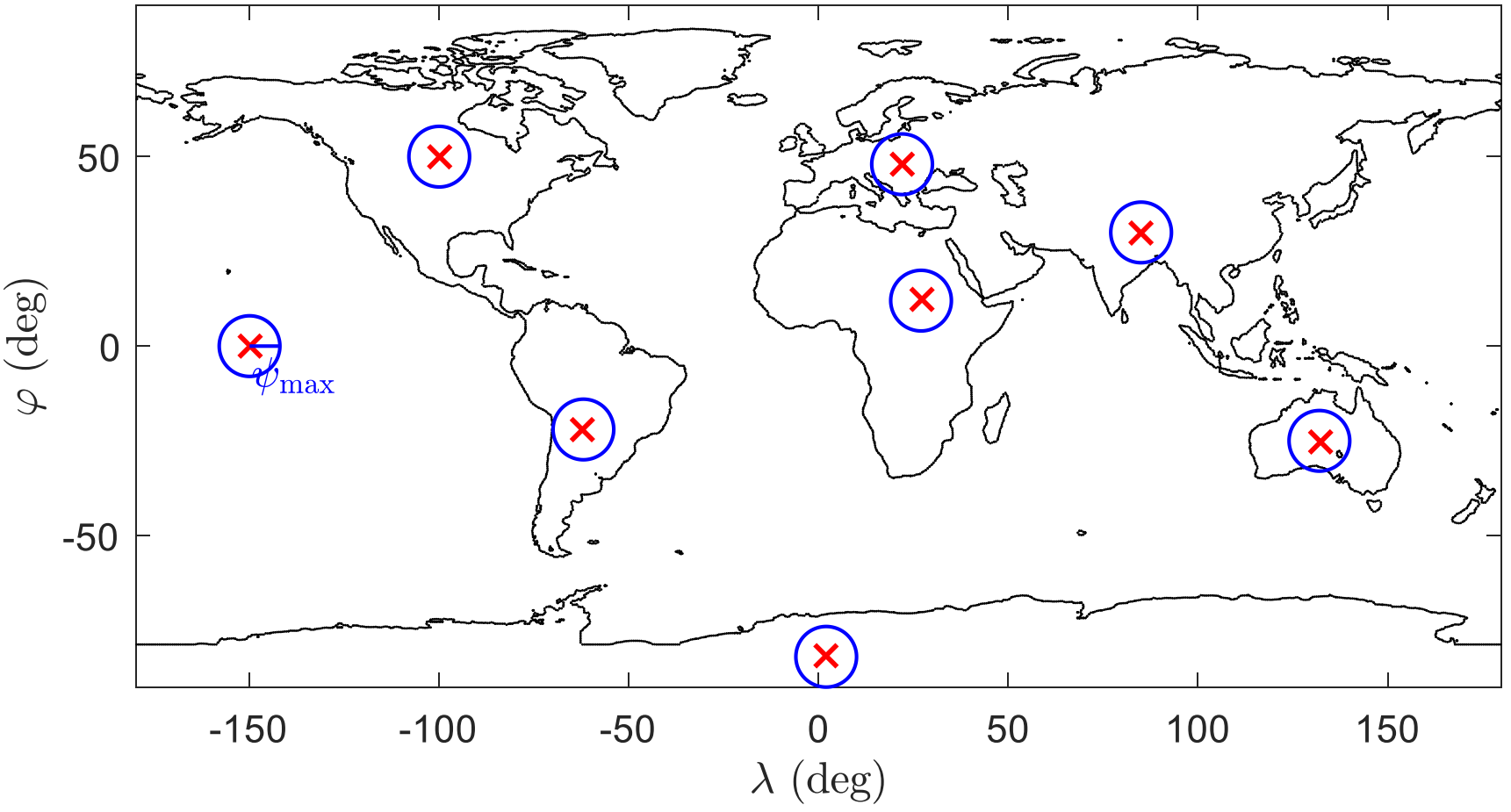
- Comparison of estimated and initial offsets

Unification of height datums
Choice of height offsets added to EGM2008 data



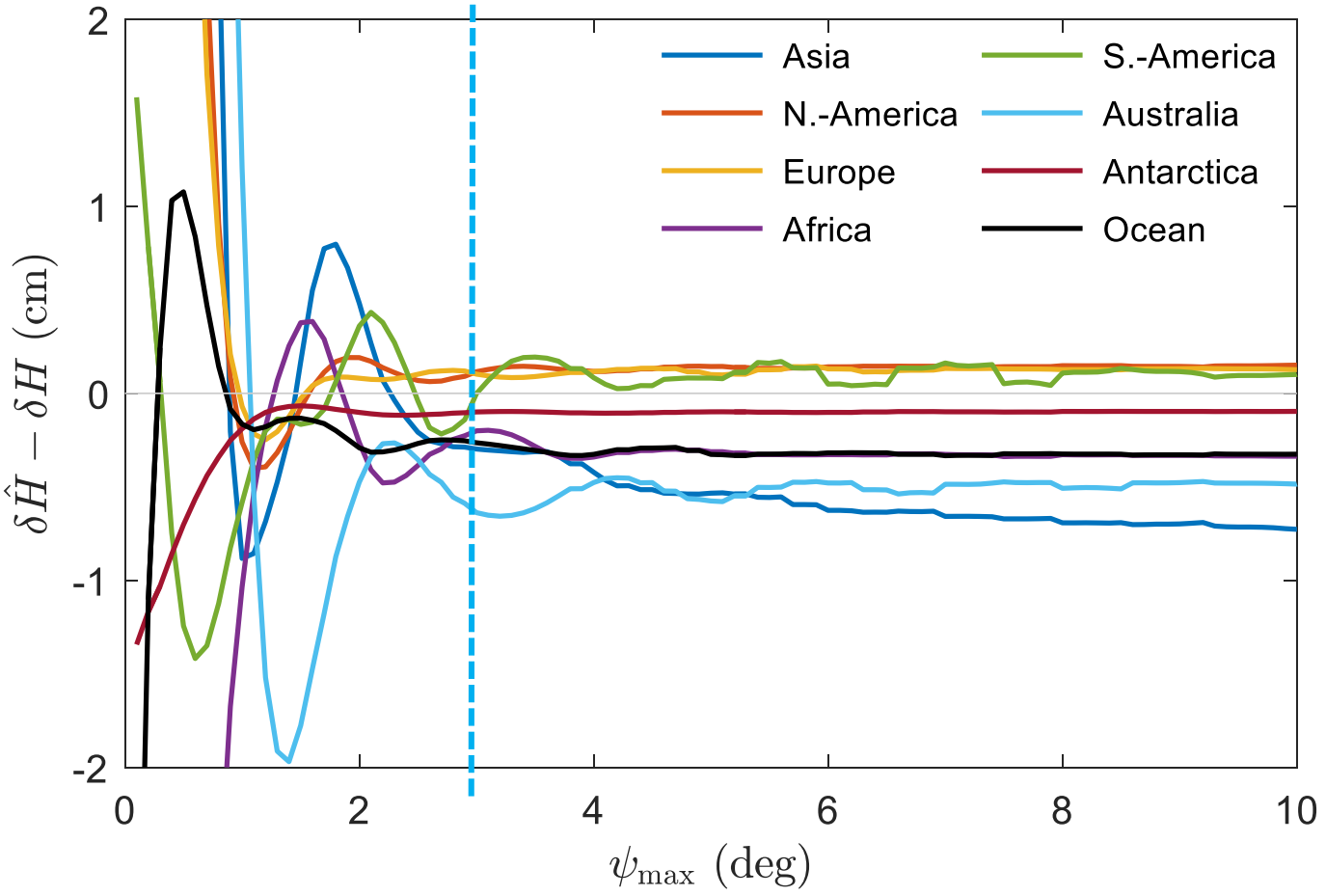
(see Grombein et al., 2016)

Unification of height datums
Enlarging the integration radii around test points



Unification of height datums
Determining height datum offsets (closed loop)

- Using GGM ($N = 200$) and modification acc. H&G with 1 computation point per datum zone



Conclusion

- Results of the simulation
  - Error of height anomalies and height datum offsets with **one computation point per zone** and use of **GGM with $N \geq 200$** and **H&G-modification**: < 1 cm at 3° integration radius

- Outlook
  - Non-linear, ellipsoidal and **topographic** effects
  - Use of **stochastic** data
    - Implementation with real data

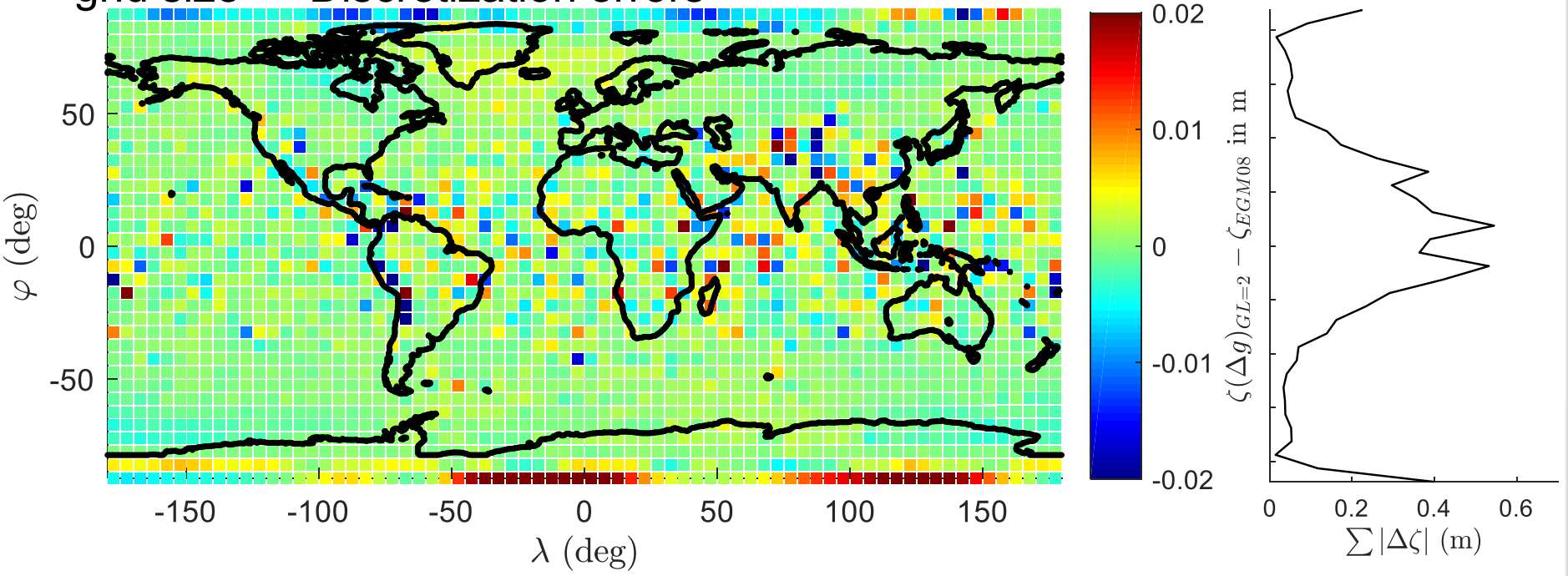
Literature

Grombein, T. et al. (2016), Height System Unification Based on the Fixed GBVP Approach. In: C. Rizos, P. Willis (Hg.), *IAG 150 Years: Proceedings of the IAG Scientific Assembly in Postdam, Germany, 2013*, S. 305–311, Springer International Publishing. http://dx.doi.org/10.1007/1345_2015_104

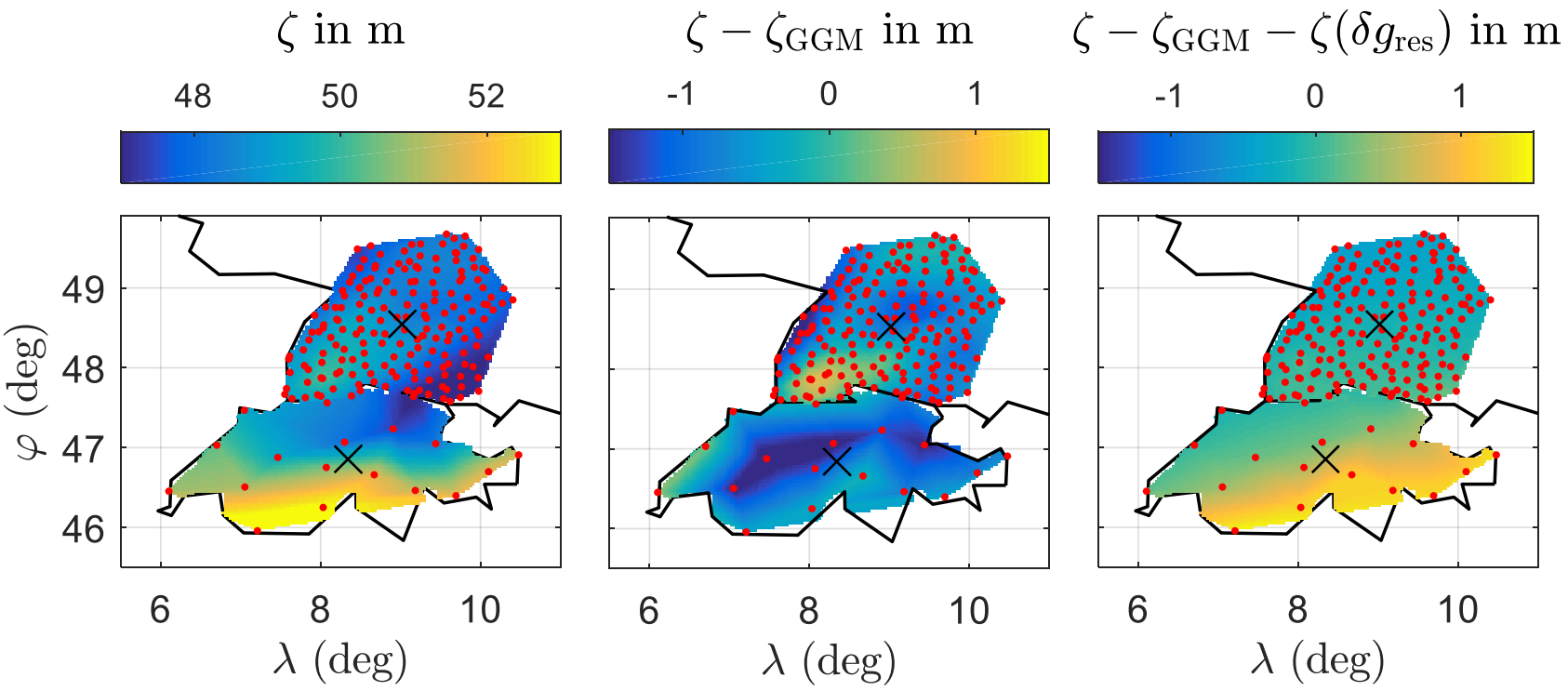
Featherstone, W.E. (2013), Deterministic, stochastic, hybrid and band-limited modifications of Hotine's integral. *Journal of Geodesy*, 87(5):487–500 http://dx.doi.org/10.1007/s00190-013-0612-9

Numerical Integration

- Difference of integrated height anomalies $\zeta = \frac{T}{\bar{\gamma}}$ and EGM08-setpoints
- Discrepancies of up to 12 cm (6 mm RMS)
- Improvement through higher order of quadrature or smaller integration grid size → Discretization errors

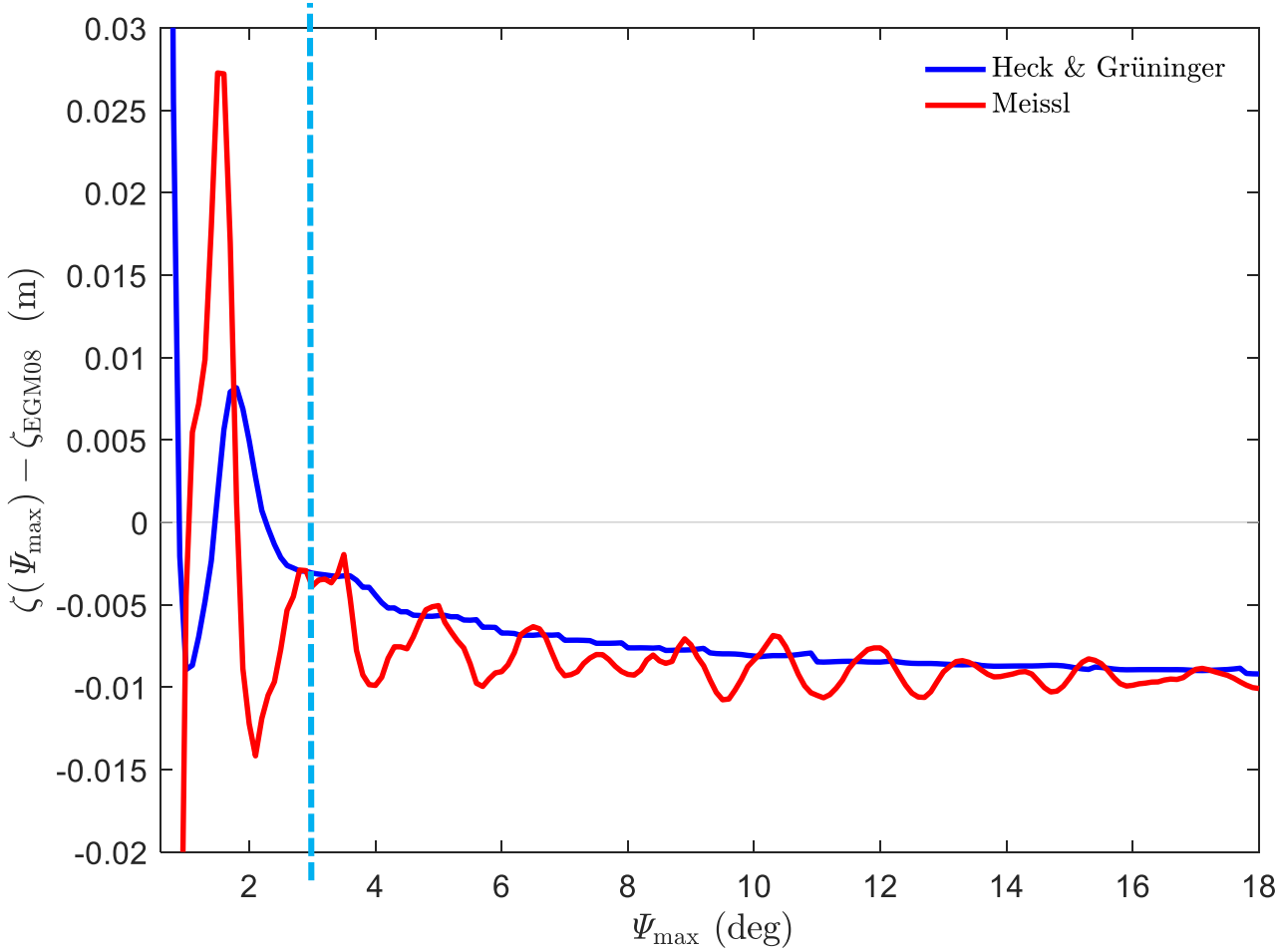


First results with real data

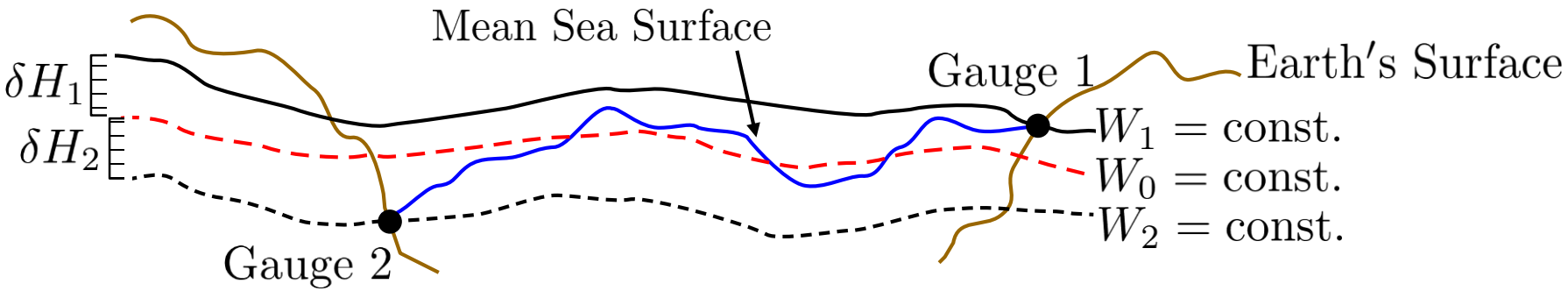


Incorporation of a geopotential model

- $(N = 200)$, test point Himalaya:

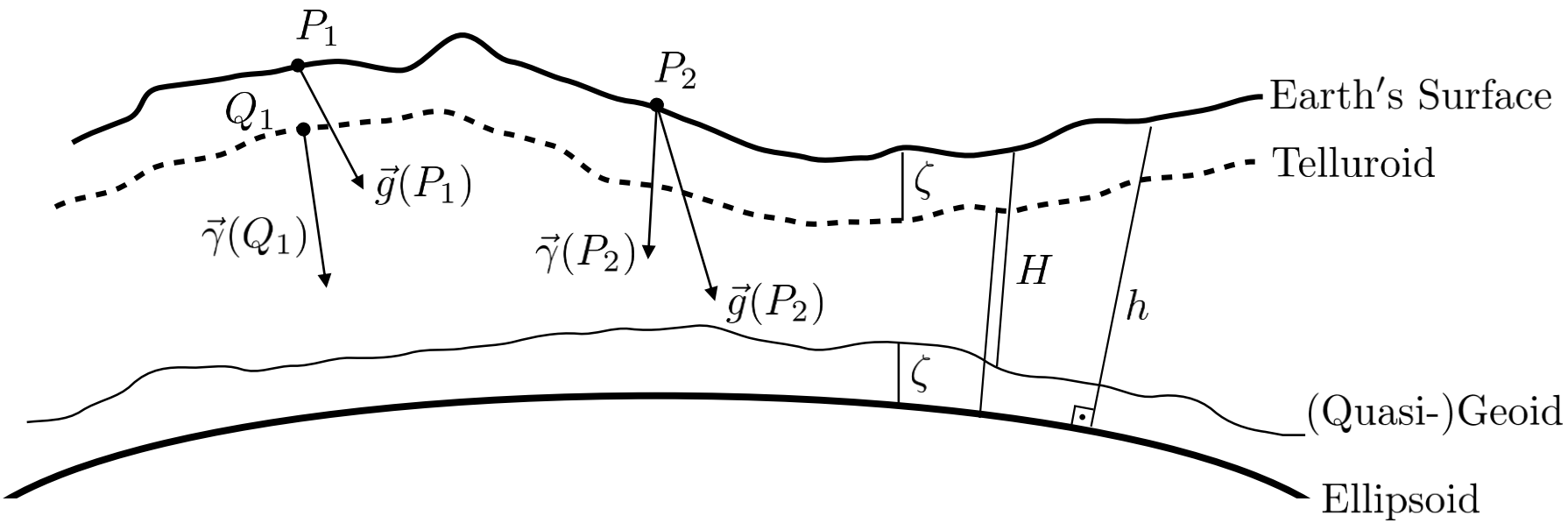


Cause of different height datums

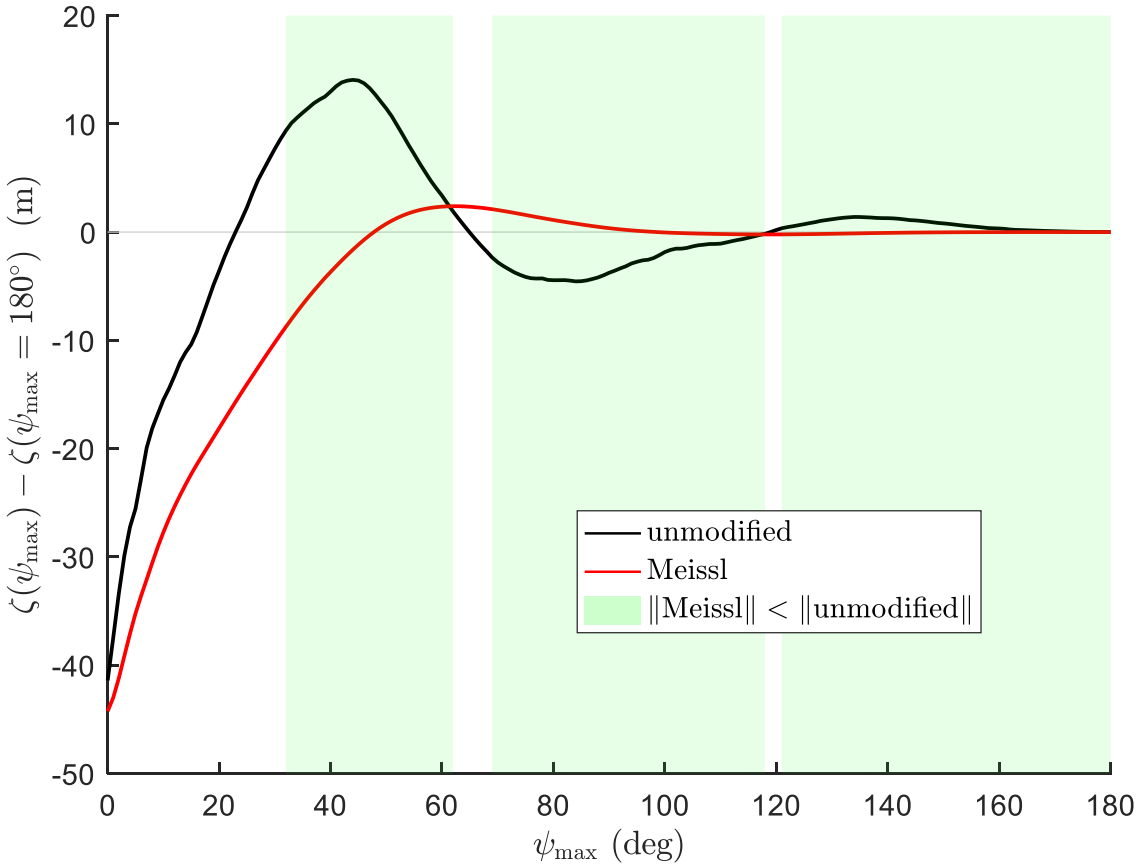


Geodetic Boundary Value Problems

Height and Gravity Definitions



Impact of the modifications on height anomaly



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- Meissl-Modification: