Weekly TRF realization from non-singular input NEQ and related distorting effects in minimally constrained solutions

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European Geosciences Union

General Assembly 2017

Vienna, Austria, 23-28 April 2017

STATEMENT OF THE PROBLEM

Terrestrial reference frames (TRFs) are often realized by space geodetic techniques via the constrained least-squares inversion of a system of normal equations (NEQ) which contains as primary unknowns the sought station coordinates over global, continental or regional networks.

This covers all usual cases of TRF realization, including: (a) epoch solutions by single network analysis or sub-network combination, (b) single-technique cumulative solutions by multi-epoch NEQ stacking, and (c) inter-technique combination solutions.

In relation to NEQ-based frame realization schemes, the following aspects should be noted:

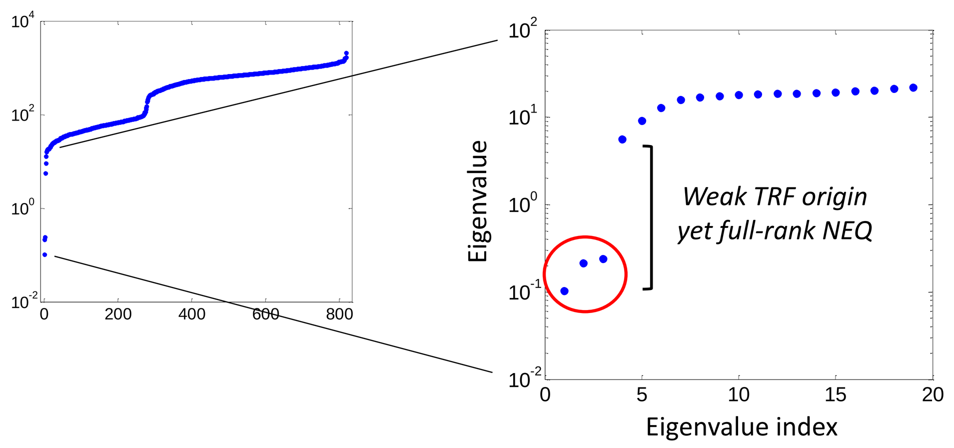
- *The input NEQ often have full rank (i.e. their normal matrix is invertible) due to datum information which is carried by various modeling choices or other software-dependent procedures during the data analysis stage.*
- *The regularity of the input NEQ may cause unwanted effects in the TRF solution since the addition of external datum constraints to a normal matrix that is already invertible can affect the geometrical (and other well estimable) characteristics of the adjusted network.*
- *As a result, the estimated station coordinates will produce a distorted TRF solution in the sense that the information content of the geodetic measurements may be altered at a significant level during the constrained NEQ inversion.*

The above aspects do not necessarily imply a "wrong" TRF solution, yet they signify **the inability to obtain a genuine distortion-free TRF solution from space geodetic data under the so-called minimal constraints.** Therefore, a reasonable concern for TRF studies is whether the regularity of the input NEQ could cause a sizeable distortion in the adjusted network even if the practically minimal constraints (e.g. *Sillard and Boucher 2001*) are applied for the external datum definition in the computed frame. An attempt to address this concern is given in the present study.

Our main objective here is to demonstrate the extent of spatial distortion in the minimally-constrained weekly solutions of the **European Permanent GNSS Network** (EPN) due to the absence of proper rank defect in the original (unconstrained) combined weekly NEQs.

INPUT NEQ DIAGNOSTICS: AN EXAMPLE WITH A WEEKLY COMBINED EPN SOLUTION

Eigenvalue analysis of the unconstrained normal matrix



A rank-deficient matrix is characterized by a cluster of small eigenvalues with a ***sizeable gap*** relative to the rest of the spectrum. If such rank deficiency cannot be numerically justified then the lower part of the spectrum can still expose the presence of ill-defined TRF parameters in the underlying NEQ.

In the above plots the 3 smallest eigenvalues reflect the weakly defined TRF origin by the fixed satellite orbits in the combined weekly NEQ of the EPN network. The gap in the lower band of the spectrum is not larger than 10^2, whereas the ratio between the maximum and minimum eigenvalues does not exceed 10^4. This implies a ***well-conditioned invertible*** normal matrix!

Reference system effect – statistical view

The datum defect of a normal matrix $\boldsymbol{N}$ can be inferred via the covariance matrix $\boldsymbol{\Sigma}=(\boldsymbol{G}\boldsymbol{N}\boldsymbol{G}^T)^{-1}$ which quantifies the so-called reference system effect, as explained by *Sillard and Boucher (JGeod, 2001)*. The matrix $\boldsymbol{G}$ is the well-known Helmert transformation matrix and it contains 7 or 14 rows (depending on the NEQ type: "static" or "kinematic") each of which corresponds to a fundamental TRF parameter.

The following table gives the reference system effect in a weekly combined (unconstrained) NEQ system of the EPN network. Note that the frame origin is resolved much less accurately than the frame orientation and scale, as it should be normally expected in such a non-global GNSS network. However, this does not mean that the normal matrix will be rank deficient (singular) in a strict numerical sense!

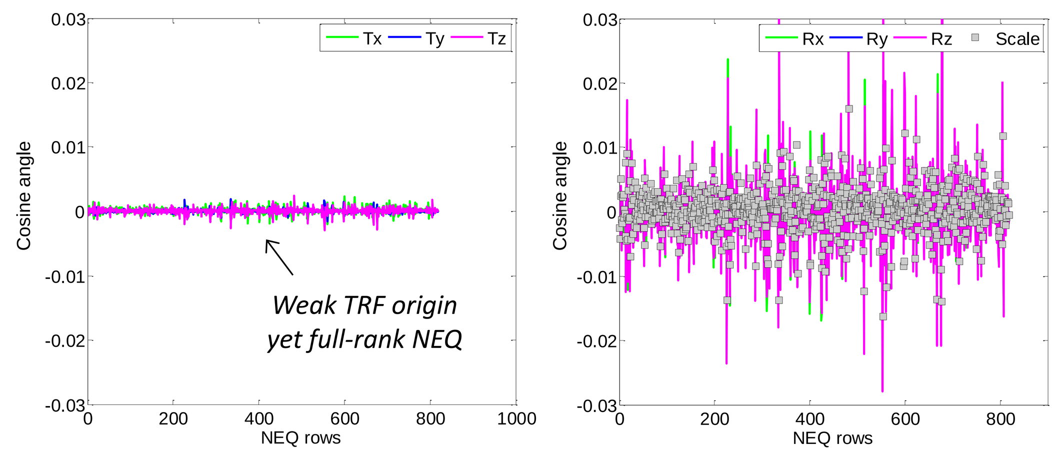
Square roots of the diagonal elements of the covariance matrix Σ

	Tx	Ty	Tz	Rx	Ry	Rz	Scale
std	17.7	15.5	20.1	5.7	7.3	4.8	4.8

All values given in cm.

The standard deviations of the TRF orientation and scale have been converted to linear units using the Earth radius value R=6378137 m.

Reference system effect – geometrical view



The datum defect of a normal matrix is related to the fact that all of its columns are orthogonal to (some or all of) the rows of the Helmert transformation matrix. Algebraically this means that the product $\boldsymbol{N}\boldsymbol{G}^T$ has one or more zero columns, depending on the number of ill-defined frame parameters in the underlying NEQ.

For numerical convenience, the aforementioned ***orthogonality check*** should be applied via the normalized matrix:

$$\boldsymbol{N}\boldsymbol{G}^T=\begin{bmatrix} \frac{\boldsymbol{n}_1^T\boldsymbol{g}_1}{\|\boldsymbol{n}_1\|\cdot\|\boldsymbol{g}_1\|} & \cdots & \frac{\boldsymbol{n}_1^T\boldsymbol{g}_7}{\|\boldsymbol{n}_1\|\cdot\|\boldsymbol{g}_7\|} \\ \frac{\boldsymbol{n}_2^T\boldsymbol{g}_1}{\|\boldsymbol{n}_2\|\cdot\|\boldsymbol{g}_1\|} & \cdots & \frac{\boldsymbol{n}_2^T\boldsymbol{g}_7}{\|\boldsymbol{n}_2\|\cdot\|\boldsymbol{g}_7\|} \\ \vdots & \vdots & \vdots \\ \frac{\boldsymbol{n}_m^T\boldsymbol{g}_1}{\|\boldsymbol{n}_m\|\cdot\|\boldsymbol{g}_1\|} & \cdots & \frac{\boldsymbol{n}_m^T\boldsymbol{g}_7}{\|\boldsymbol{n}_m\|\cdot\|\boldsymbol{g}_7\|} \end{bmatrix}$$

→ **Cosine angles between the columns n_i of the normal matrix and the rows g_k of the Helmert transformation matrix.**

An example of this matrix is given in the above plots (left plot shows the first 3 columns, right plot shows the rest 4 columns).

(*) This example refers to the original (unconstrained) normal matrix which is extracted from a weekly SINEX file in the EPN network. The weekly SINEX file corresponds to the combined solution for the period 1-7/11/2015 (ftp://epncb.oma.be/pub/product/combin/1869/EUR18697.snx).

SPATIAL DISTORTIONS OF MINIMALLY-CONSTRAINED WEEKLY SOLUTIONS IN THE EPN NETWORK

Detection methodology

- INPUT: (unconstrained) weekly combined NEQ of the EPN network.
- SOLUTION I: weekly datum-free solution computed by regular NEQ inversion **without adding any external constraints**.
- SOLUTION II: weekly minimally-constrained solution computed by adding **NNT constraints** wrt. IGb08 over a subset of EPN stations.
- A diagonal weight matrix is used for the NNT constraints, which is tuned to an accuracy level of 10^{-5} m towards the TRF origin fixation.

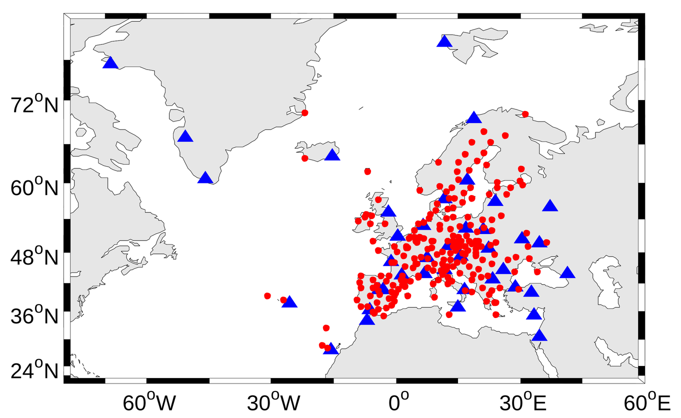


Fig. 1: The EPN GNSS network. The blue triangles represent the used reference stations for the NNT-based weekly solutions.

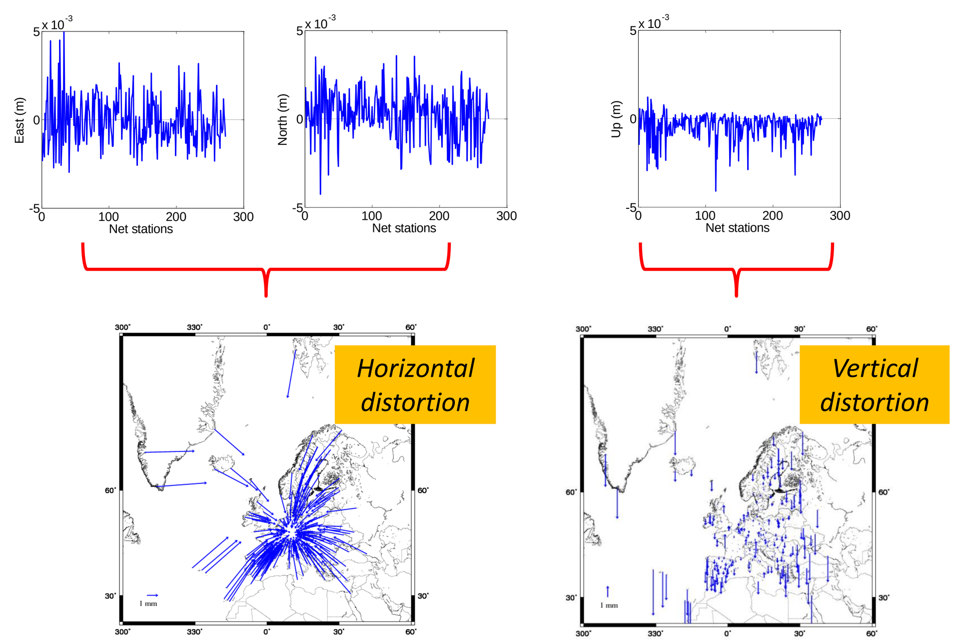


Fig. 2: Post-fit residuals and their spatial behavior after applying a shift/rotation Helmert transformation between the datum-free and NNT weekly solutions in the EPN network. The input NEQ refer to the time period 1-7/11/2015.

Detection methodology (cont'd)

- Perform a least-squares fit between SOLUTION I and SOLUTION II using the Helmert transformation model.
- The transformation residuals reflect the weekly EPN distortion **due to the pseudo-minimal character** of the external NNT constraints (see example above).
- The estimated transformation parameters also contain a part of the produced distortion on the well estimable TRF parameters.

Results (sample)

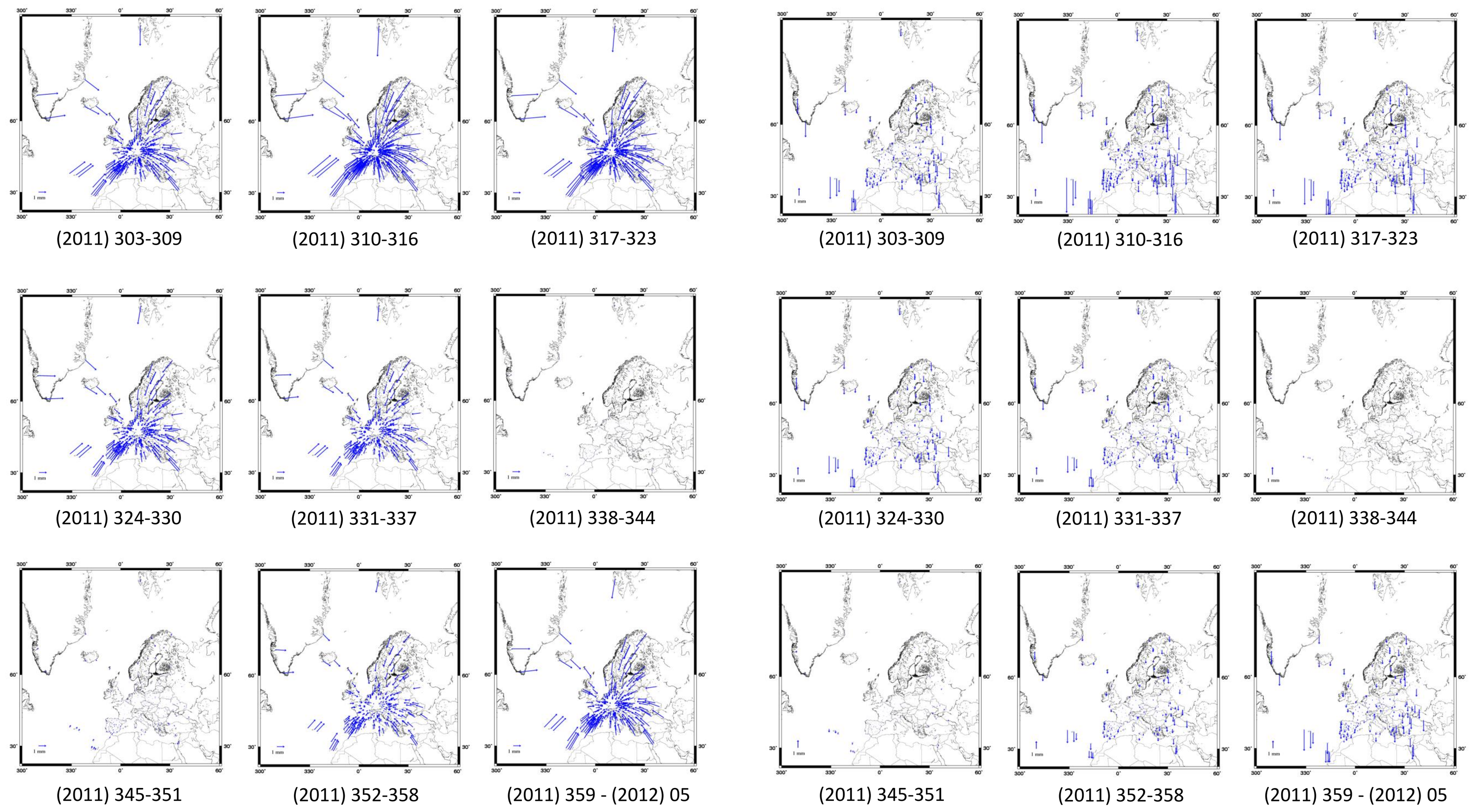


*Fig. 3: Maps of horizontal and vertical post-fit residuals after applying a shift/rotation Helmert transformation between the **datum-free and NNT weekly solutions** in the EPN network. The input unconstrained NEQ refer to different consecutive weeks in 2011, as specified in each plot. Note that similar results were obtained by using several other weekly EPN SINEX files throughout different years.*

OUTLOOK OF RESULTS

The addition of "minimal constraints" to input NEQ without proper rank defect can cause unwanted distorting effects in the computed TRF solution.

In the case of the EPN network such distortions can reach **several mm** and they will affect both the horizontal and vertical components of the weekly combined solutions which are obtained by the external NNT conditions.

The major part of the weekly EPN **horizontal distortions** seems to be equivalent to a scale-like bias of a few ppb. On the other hand, the **vertical distortions** do not follow a similar behavior and they may reach up to 3-4 mm without being able to be absorbed by a uniform spatial re-scaling.

The orientation of the EPN weekly combined solutions is also affected by the applied NNT conditions! In fact, rotational differences in the order of 0.1 mas were detected between the datum-free and NNT-based weekly solutions.

To avoid this kind of distorting effects in minimally-constrained solutions, **the input NEQ should be properly filtered** by removing all datum information for frame parameters that will be handled by external datum conditions during the NEQ inversion (*see Kotsakis and Chatzinikos, JGeod 2017, in press*).