Viscoelastic Waves Simulation in a Blocky Medium with Fluid-Saturated Interlayers Using High-Performance Computing

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The Grandfather

The First Pillar

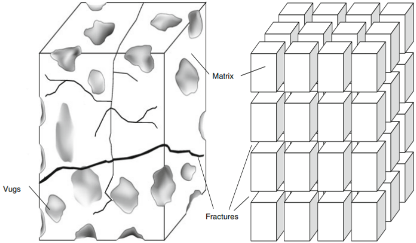
The Feathers

Contents

Blocky model of fractured reservoirs

The faults and/or filed fractures in the reservoir introduce a network, communicate hydraulically between each other locally and globally, and provide overall conductivity (permeability) of the reservoir, and the matrix provides overall storage capacity (porosity).

Dual-porosity reservoir model

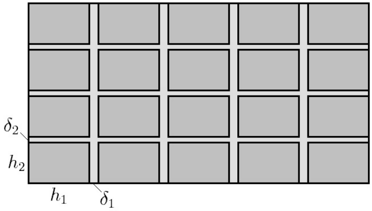


Fractured reservoir   Sugar cube representation

The picture is taken from

Warren J.E. and Root P.J. **The behavior of naturally fractured reservoirs**
SPE J., **3**, 245–255, 1963.

Equations of elastic blocks and elastic interlayers



Scheme of a blocky medium

A motion of each block is defined by the system of equations of a homogeneous isotropic elastic medium:

$$
\begin{aligned}
\rho\,\dot v_1 &= \sigma_{11,1} + \sigma_{12,2}\\
\rho\,\dot v_2 &= \sigma_{12,1} + \sigma_{22,2}\\
\dot\sigma_{11} &= \rho\,c_1^2\big(v_{1,1} + v_{2,2}\big) - 2\,\rho\,c_2^2\,v_{2,2}\\
\dot\sigma_{22} &= \rho\,c_1^2\big(v_{1,1} + v_{2,2}\big) - 2\,\rho\,c_2^2\,v_{1,1}\\
\dot\sigma_{12} &= \rho\,c_2^2\big(v_{2,1} + v_{1,2}\big)
\end{aligned}
$$

Elastic interlayer between the horizontally located nearby blocks is described by the system of equations:

$$
\rho'\,\frac{\dot v_1^+ + \dot v_1^-}{2} = \frac{\sigma_{11}^+ - \sigma_{11}^-}{\delta_1}, \qquad \frac{\dot\sigma_{11}^+ + \dot\sigma_{11}^-}{2} = \rho'\,{c_1'}^2\,\frac{v_1^+ - v_1^-}{\delta_1}
$$

$$
\rho'\,\frac{\dot v_2^+ + \dot v_2^-}{2} = \frac{\sigma_{12}^+ - \sigma_{12}^-}{\delta_1}, \qquad \frac{\dot\sigma_{12}^+ + \dot\sigma_{12}^-}{2} = \rho'\,{c_2'}^2\,\frac{v_2^+ - v_2^-}{\delta_1}
$$

Elastic interlayer between the vertically located nearby blocks is modeled using similar system:

$$
\rho'\,\frac{\dot v_2^+ + \dot v_2^-}{2} = \frac{\sigma_{22}^+ - \sigma_{22}^-}{\delta_2}, \qquad \frac{\dot\sigma_{22}^+ + \dot\sigma_{22}^-}{2} = \rho'\,{c_1'}^2\,\frac{v_2^+ - v_2^-}{\delta_2}
$$

$$
\rho'\,\frac{\dot v_1^+ + \dot v_1^-}{2} = \frac{\sigma_{12}^+ - \sigma_{12}^-}{\delta_2}, \qquad \frac{\dot\sigma_{12}^+ + \dot\sigma_{12}^-}{2} = \rho'\,{c_2'}^2\,\frac{v_1^+ - v_1^-}{\delta_2}
$$

The case of elastic-plastic interlayers

To take into account the plasticity, constitutive equations of the vertical elastic interlayer are replaced by the variational inequality:

$$(\delta\sigma_{11}^{+} + \delta\sigma_{11}^{-})\,\dot\varepsilon_{11}^{p} + (\delta\sigma_{12}^{+} + \delta\sigma_{12}^{-})\,\dot\varepsilon_{12}^{p} \leqslant 0$$

$\delta\sigma_{jk}^{\pm} = \tilde\sigma_{jk}^{\pm} - \sigma_{jk}^{\pm}$ – variations of stresses

$\dot\varepsilon_{11}^{p} = \dfrac{v_1^{+} - v_1^{-}}{\delta_1} - \dfrac{\dot\sigma_{11}^{+} + \dot\sigma_{11}^{-}}{2\,\rho' {c_1'}^2}, \quad \dot\varepsilon_{12}^{p} = \dfrac{v_2^{+} - v_2^{-}}{\delta_1} - \dfrac{\dot\sigma_{12}^{+} + \dot\sigma_{12}^{-}}{2\,\rho' {c_2'}^2}$ – plastic strain rates

The actual stresses $\sigma_{jk}^{\pm}$ and variable stresses $\tilde\sigma_{jk}^{\pm}$ are subject to the constraint in the form:

$$f\left(\frac{\tilde\sigma_{11}^{+} + \tilde\sigma_{11}^{-}}{2}, \frac{\tilde\sigma_{12}^{+} + \tilde\sigma_{12}^{-}}{2}\right) \leqslant \tau(\chi)$$

τ – the material yield point of interlayers, χ – a material parameter (or set of parameters) of hardening
$f(\sigma_n, \sigma_\tau)$ – the equivalent stress function, in which arguments are normal and tangential stresses

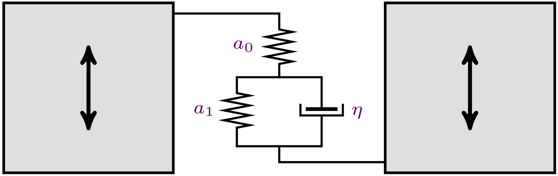
The simplest form of the constraint for a microfractured medium is as follows:

$$|\sigma_\tau| \leqslant \tau_s - k_s\,\sigma_n \qquad (\tau_s \text{ and } k_s \text{ – the material parameters})$$

Constitutive equations of the horizontal elastic-plastic interlayer are formulated in a similar way

Poynting–Thomson viscoelastic model

To describe the viscous dissipative effects in the interlayers under the shear waves propagation, which influence the solution in the long run, the Poynting–Thomson model of a viscoelastic medium is used.



Poynting–Thomson's rheological scheme

Hooke's law for elastic element: $\varepsilon'_{12} = a_0\,(\sigma_{12}^+ + \sigma_{12}^-)/2,\;\; \varepsilon''_{12} = a_1 s_{12}$

Newton's law for viscous element: $\eta\,\dot\varepsilon''_{12} = (\sigma_{12}^+ + \sigma_{12}^-)/2 - s_{12}$ Total strain: $\varepsilon_{12} = \varepsilon'_{12} + \varepsilon''_{12}$

Constitutive equations of the interlayer:

$$a_0\,\frac{\dot\sigma_{12}^+ + \dot\sigma_{12}^-}{2} + a_1\,\dot s_{12} = \frac{v_2^+ - v_2^-}{\delta_1}, \qquad \frac{\sigma_{12}^+ + \sigma_{12}^-}{2} = s_{12} + \eta\,a_1\,\dot s_{12}$$

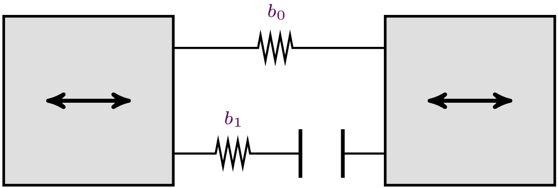
Energy balance equation:

$$\frac{\sigma_{12}^+ + \sigma_{12}^-}{2}\,\frac{v_2^+ - v_2^-}{\delta_1} = \dot W + \eta\,a_1^2\,\dot s_{12}^2, \quad 2\,W = a_0\,\frac{(\sigma_{12}^+ + \sigma_{12}^-)^2}{4} + a_1\,s_{12}^2$$

according to which the power of internal stresses in the interlayer is the sum of the reversible elastic strain power and the power of the viscous energy dissipation

Model of porous interlayers

The longitudinal deformation of the interlayers is described on the basis of a complicated version of the porous elastic model, which takes into account the nonlinear threshold behavior of a material with the strength increasing during the collapse of pores.



Rheological scheme of a porous interlayer

Total strain: $\varepsilon_{11} = \sigma'_{11}/b_1 + \theta_1 - \theta_0$

$\sigma'_{11} \leqslant 0$ – stress in a rigid contact, $\theta_0 > 0$ and $\theta_1 \geqslant 0$ – initial and current porosity values

Governing relationships of a rigid contact: $(\tilde\sigma_{11} - \sigma'_{11})\,\theta_1 \leqslant 0, \quad \tilde\sigma_{11},\, \sigma'_{11} \leqslant 0$

$\sigma'_{11} = b_1\,\pi(\theta_0 + \varepsilon_{11}),\ \ \pi(\theta) = \min(\theta, 0)$ – projection onto the non-positive semi-axis

Constitutive equations of the interlayer including the equation for porosity:

$$\dot\varepsilon_{11} = \frac{v_1^+ - v_1^-}{\delta_1}, \quad \frac{\sigma_{11}^+ + \sigma_{11}^-}{2} = b_0\,\varepsilon_{11} + b_1\,\pi(\theta_0 + \varepsilon_{11}), \quad \theta_1 = \theta_0 + \varepsilon_{11} - \pi(\theta_0 + \varepsilon_{11})$$

The energy balance equation: $\dfrac{\sigma_{11}^+ + \sigma_{11}^-}{2}\,\dot\varepsilon_{11} = \dot W, \quad 2\,W = b_0\,\varepsilon_{11}^2 + b_1\,\pi^2(\theta_0 + \varepsilon_{11})$

Modified Biot's model

Under numerical modeling of the wave motion in a blocky medium containing fluid-saturated porous interlayers, a version of the model is applied based on Biot's approach.

Kinetic energy related to the initial unit of a volume of the horizontal interlayers:

$$2\,T = \rho_s\,\frac{(v_1^+ + v_1^-)^2}{4} + \rho_a\Big(\frac{v_1^+ + v_1^-}{2} - w_1\Big)^2 + (\rho_s + \rho_f)\,\frac{(v_2^+ + v_2^-)^2}{4} + \rho_f\,w_1^2$$

ρ_s, ρ_f – partial densities of a solid skeleton and a liquid phase in interlayers at the initial moment of time
ρ_a – density of additional mass used to take into account the mutual influence of fluid and skeleton in the case of relative motion, w_1 – absolute velocity of the fluid motion

Equations describing skeleton motion in the direction of longitudinal axis x_1:

$$(\rho_s + \rho_a)\,\frac{\dot v_1^+ + \dot v_1^-}{2} - \rho_a\,\dot w_1 = \frac{\sigma_{12}^+ - \sigma_{12}^-}{\delta_2}$$

$$a_0\,\frac{\dot\sigma_{12}^+ + \dot\sigma_{12}^-}{2} + a_1\,\dot s_{12} = \frac{v_1^+ - v_1^-}{\delta_2}, \qquad \frac{\sigma_{12}^+ + \sigma_{12}^-}{2} = s_{12} + \eta\,a_1\,\dot s_{12}$$

Equations describing joint motion of the solid and liquid phase in the direction of transverse axis x_2:

$$(\rho_s + \rho_f)\,\frac{\dot v_2^+ + \dot v_2^-}{2} = \frac{\sigma_{22}^+ - \sigma_{22}^-}{\delta_2}, \quad \dot\varepsilon_{22} = \frac{v_2^+ - v_2^-}{\delta_2}, \quad \frac{\dot\sigma_{22}^+ + \dot\sigma_{22}^-}{2} = b_0\,\dot\varepsilon_{22} + b_1\,\dot\pi(\theta_0 + \varepsilon_{22}) + b_s\,w_{1,1}$$

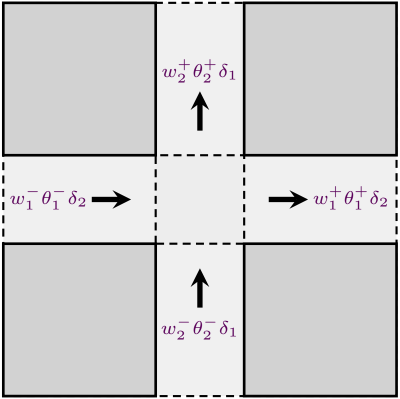
Equations describing the fluid motion along the interlayer:

$$(\rho_f + \rho_a)\,\dot w_1 - \rho_a\,\frac{\dot v_1^+ + \dot v_1^-}{2} = s_{11,1}, \qquad \dot s_{11} = b_f\,w_{1,1} + b_s\,\dot\varepsilon_{22}$$

$s_{11} = -p\,\theta$ – normal stress in the liquid phase, p – value of the pore pressure
θ – momentary porosity value, b_s and b_f – elastic moduli characterizing the interaction in the system "solid skeleton–fluid"

Kirchhoff's law for nodes

To solve the systems numerically, the computational algorithm is developed. The Godunov gap decay scheme is applied at the stage of approximation of the equations for velocity w_1 and stress s_{11} in a fluid.



Scheme of flows interaction

At junction zones of the horizontal and vertical interlayers, the internal boundary conditions are set.

They result from Kirchhoff's law for the fluid flow:

$$w_1^+ \, \theta_1^+ \, \delta_2 + w_2^+ \, \theta_2^+ \, \delta_1 = w_1^- \, \theta_1^- \, \delta_2 + w_2^- \, \theta_2^- \, \delta_1$$

and the dynamic equations:

$$s_{11}^\pm = -p \, \theta_1^\pm, \quad s_{22}^\pm = -p \, \theta_2^\pm$$

considering the pressure equality at a junction.

$\theta_1^\pm$ and $\theta_2^\pm$ – porosities in the horizontal and vertical interlayers

In this formulation of the boundary conditions at the junctions, the power balance equation is fulfilled:

$$s_{11}^+ \, w_1^+ \, \delta_2 + s_{22}^+ \, w_2^+ \, \delta_1 - s_{11}^- \, w_1^- \, \delta_2 - s_{22}^- \, w_2^- \, \delta_1 = 0$$

by which the thermodynamic consistency of equations in the interlayers with governing equations in the blocks can be proved.

Two-cyclic splitting

On the basis of discrete analogs of the equations in blocks and interlayers, the parallel computational algorithm is developed for the analysis of waves propagation in a blocky media with porous fluid-saturated interlayers on supercomputers of the cluster architecture. A suitable two-cyclic method of splitting with respect to the spatial variables is applied, which has high accuracy and permits the efficient parallelization of computations.

Governing equations in blocks and interlayers can be provided in the form of symbolic evolution equation:

$$\dot{U} = A_1(U) + A_2(U)$$

A_1 and A_2 – nonlinear differential-difference operators, simulating 1D motion of a blocky medium in the direction of the coordinate axes x_1 and x_2, U – vector–function of unknown quantities which includes the projection of the velocity vector and the stress tensor in blocks and interlayers

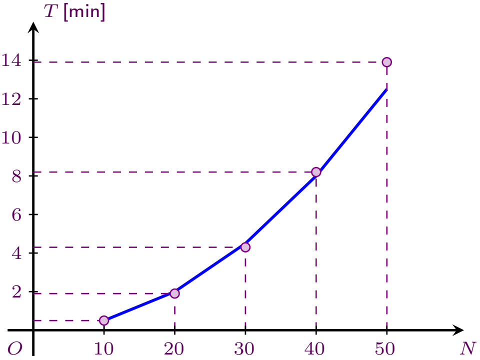
In such notations the method of splitting on the time interval $(t_0, t_0 + \Delta t)$ includes four steps: the step of solving 1D equation in the x_1 direction on the interval $(t_0, t_0 + \Delta t/2)$, a similar step of solving the equation in the x_2 direction, the step of recomputation in the x_2 direction on the interval $(t_0 + \Delta t/2, t_0 + \Delta t)$ and the step of recomputation in the x_1 direction on the same interval:

$$\begin{aligned}
&\dot{U}^{(1)} = A_1(U^{(1)}), \quad U^{(1)}(t_0) = U(t_0)\\
&\dot{U}^{(2)} = A_2(U^{(2)}), \quad U^{(2)}(t_0) = U^{(1)}(t_0 + \Delta t/2)\\
&\dot{U}^{(3)} = A_2(U^{(3)}), \quad U^{(3)}(t_0 + \Delta t/2) = U^{(2)}(t_0 + \Delta t/2)\\
&\dot{U}^{(4)} = A_1(U^{(4)}), \quad U^{(4)}(t_0 + \Delta t/2) = U^{(3)}(t_0 + \Delta t)
\end{aligned}$$

The solution at the time instant $t_0 + \Delta t$ equals to $U(t_0 + \Delta t) = U^{(4)}(t_0 + \Delta t)$

Efficiency of parallelization

Described computational algorithm is implemented as the parallel program for analysis of the waves propagation processes in blocky media under external dynamic loads on multiprocessor computer systems of the cluster architecture. The parallelization is performed on the basis of domain decomposition – each processor of a cluster expects a separate chain of blocks including the boundary interlayers in the horizontal direction. The programming language is Fortran, and the message passing interface (MPI) library is used.



Dependence of the runtime T on the linear dimension N of a grid in blocks
(circle points – actual computational time, solid line – semi-theoretical computational time)

Instant rotation of the central block in the rock mass

The developed parallel program is applied to solve a series of problems related to the waves propagation in a blocky medium under concentrated loads. The problem, in which the boundary effects around a blocky medium are absent and the initial data for velocities correspond to the rotation of the central block around the mass center at the assigned angular velocity ω_0:

$$v_1 = -\omega_0\Big(x_2 - \frac{h_2}{2}\Big), \quad v_2 = \omega_0\Big(x_1 - \frac{h_1}{2}\Big), \quad 0 \leqslant x_k \leqslant h_k \quad (k = 1, 2)$$

is solved numerically. Initial stresses in the entire rock mass and initial velocities in the blocks, except for the central block, are zero.

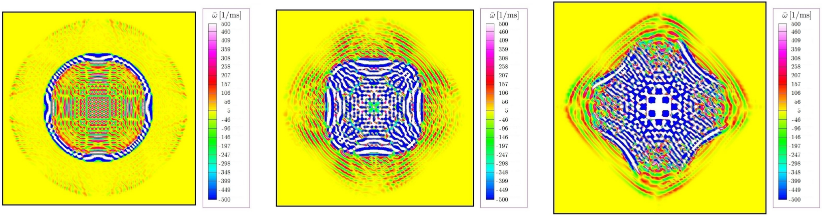
Averaged angular velocities and tangential stresses:

$$\bar{\omega} = \int\limits_0^{h_2} \frac{v_2(h_1, x_2, t) - v_2(0, x_2, t)}{2\, h_1 h_2}\, dx_2 - \int\limits_0^{h_1} \frac{v_1(x_1, h_2, t) - v_1(x_1, 0, t)}{2\, h_1 h_2}\, dx_1$$

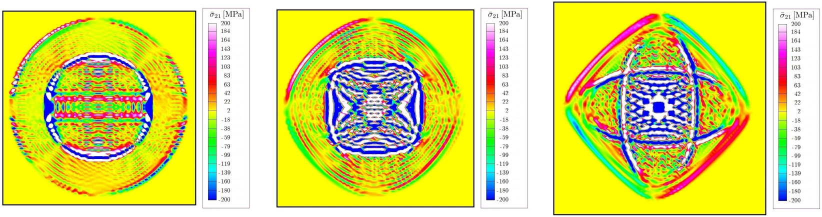
$$\bar{\sigma}_{12} = \int\limits_0^{h_1} \frac{\sigma_{12}(x_1, h_2, t) + \sigma_{12}(x_1, 0, t)}{2\, h_1}\, dx_1, \quad \bar{\sigma}_{21} = \int\limits_0^{h_2} \frac{\sigma_{12}(h_1, x_2, t) + \sigma_{12}(0, x_2, t)}{2\, h_2}\, dx_2$$

Instant rotation of the central block in the rock mass

$\delta = 0.1\,\text{mm}$ $\delta = 1\,\text{mm}$ $\delta = 5\,\text{mm}$



Level curves of the angular velocity $\bar{\omega}$ depending on the thickness of interlayers



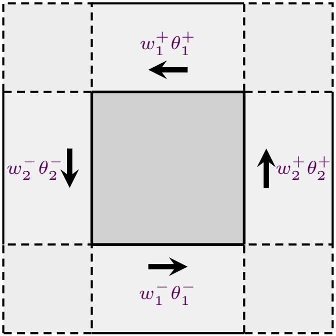
Level curves of the tangential stress $\bar{\sigma}_{21}$ depending on the thickness of interlayers

Size of each block is 50 mm × 50 mm

Instant rotation of the central block in the rock mass

The circulation C of a fluid around blocks is calculated by the formula:

$$C = \frac{\delta_1}{h_2} \int_0^{h_2} (w_2^+ \theta_2^+ - w_2^- \theta_2^-)\, dx_2 - \frac{\delta_2}{h_1} \int_0^{h_1} (w_1^+ \theta_1^+ - w_1^- \theta_1^-)\, dx_1$$

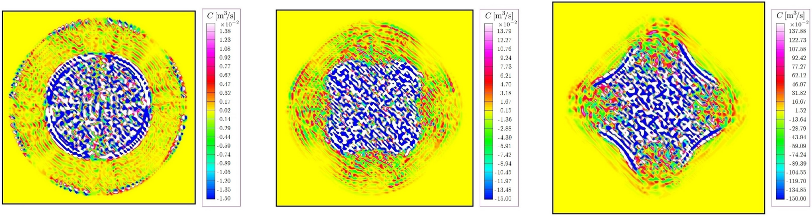


Circulation of a flow around the block

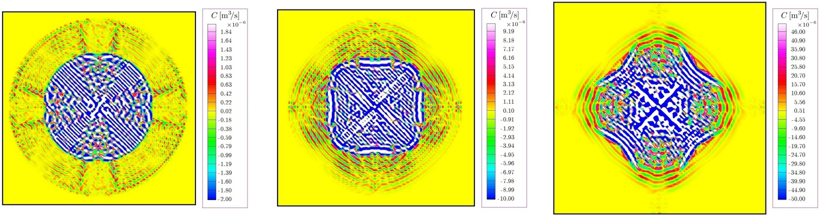
Instant rotation of the central block in the rock mass

$\delta = 0.1\,\mathrm{mm}$ $\delta = 1\,\mathrm{mm}$ $\delta = 5\,\mathrm{mm}$

The case of intensive load (with pore collapse)



Level curves of the fluid circulation C depending on the thickness of interlayers

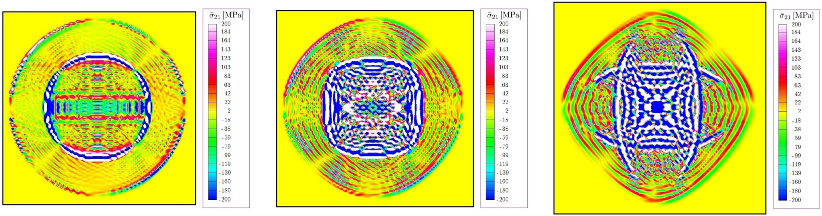


The case of small load (without pore collapse)

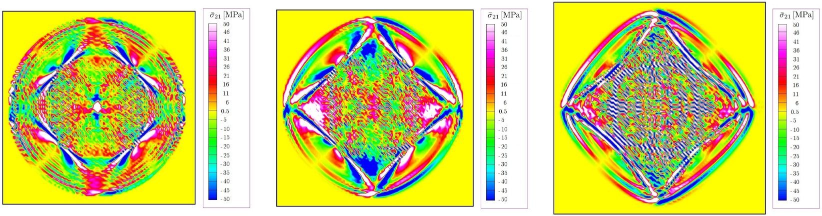
Instant rotation of the central block in the rock mass

$\delta = 0.1\,\mathrm{mm}$ $\delta = 1\,\mathrm{mm}$ $\delta = 5\,\mathrm{mm}$

The case of elastic interlayers



Level curves of the tangential stress $\bar{\sigma}_{21}$ depending on the thickness of interlayers



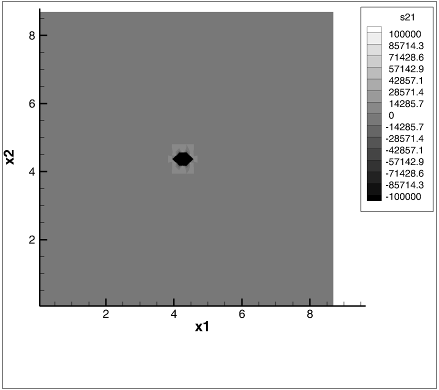
The case of elastic-plastic interlayers

Instant rotation of the central block in the rock mass

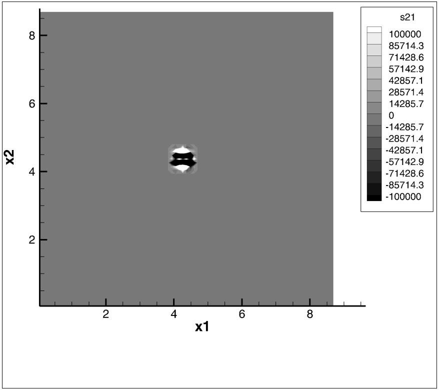
Animations

The case of elastic interlayers

Different impedances of blocks and interlayers



The same impedances of blocks and interlayers



$\delta = 8\,\mathrm{mm}$

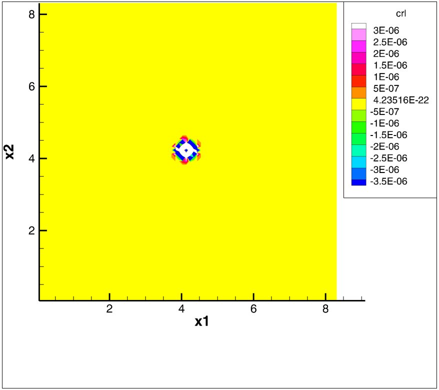
Level curves of the tangential stress $\bar{\sigma}_{21}$

Instant rotation of the central block in the rock mass

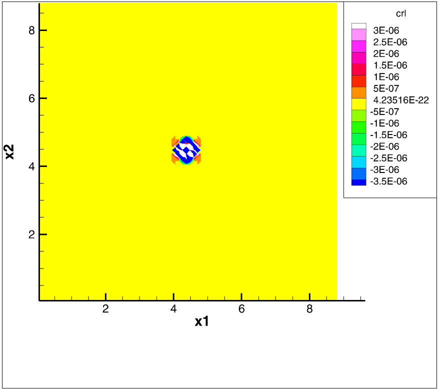
Animations

The case of fluid-saturated interlayers

$\delta = 2\,\mathrm{mm}$



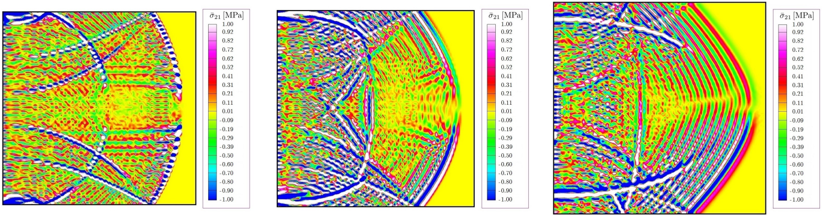
$\delta = 8\,\mathrm{mm}$



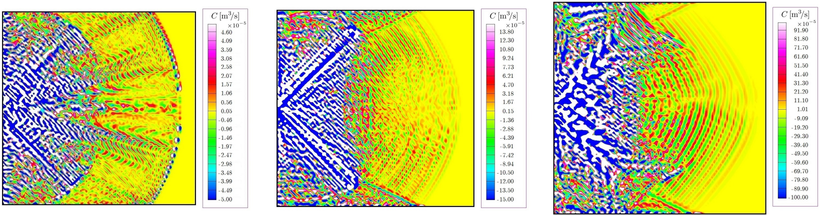
Level curves of the fluid circulation C around blocks

Concentrated rotational moment at the boundary

$\delta = 0.1\,\mathrm{mm}$ $\delta = 1\,\mathrm{mm}$ $\delta = 5\,\mathrm{mm}$



Level curves of the tangential stress $\bar{\sigma}_{21}$ depending on the thickness of interlayers



Level curves of the fluid circulation C depending on the thickness of interlayers

Conclusions

- ✓ The equations of a blocky medium with elastic blocks and interlayers, that have different mechanical properties, are proposed.
- ✓ The Poynting–Thomson rheological model, which allows to take into account viscous deformation, is used for describing the transverse waves propagation.
- ✓ The model of a porous medium is applied for the description of longitudinal waves.
- ✓ Thermodynamic consistence of the equations in interlayers with the system in blocks guarantees fulfillment of the energy conservation law for a blocky medium.
- ✓ The numerical algorithm for solving proposed system is constructed and tested.
- ✓ The parallel program system is worked out, using the MPI technology.
- ✓ By means of this software, the nonlinear wave processes in the case of initial rotation of the central block in a rock mass as well as in the case of concentrated couple stress load, applied at the boundary of a rock mass, are analyzed.

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Thank you for your attention!