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Modeling viscoelastic deformation of the earth due to surface loading by commercial finite element package - ABAQUS

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Motivation

Wu (2004) demonstrated that viscoelastic deformation of the earth due to surface loading can be modeled by ABAQUS, using an iterative stress transformation scheme. Benchmark tests confirmed that this method works extremely well on incompressible earth model. Bangtsson & Lund (2008), however, showed that if material is compressible, constitutive relation will also be transformed. So the ABAQUS method proposed by Wu (2004) is inadequate to model compressive deformation. Numerical experiments are carried out to find a new scheme without relying on stress transform completely. The proposed scheme can model the elastic deformation for self-gravitating compressible Earth accurately but not the viscoelastic response for large time. Validation test results of incompressible case are revealed here, where analytical solution is obtained by normal mode method.

Model

For incompressible material, the surface loading problem is governed by the perturbed momentum balance equation and Laplace's equation of gravity:

$$\nabla \cdot \boldsymbol{\sigma} - \nabla(\rho_0 g_0 u_r) - \rho_0 \nabla \phi_1 = 0 \qquad (1)$$

$$\nabla^2 \phi_1 = 0 \qquad (2)$$

Eq.(1) is coupled with Eq.(2) because the body force terms contain the unknown radial displacement u_r and perturbed gravitational potential ϕ_1 that are needed to be solved.

Material Parameters: The tested model consist of an incompressible uniform mantle overlying an inviscid fluid core.

	Core	Mantle
Radius (km)	3485.5	6371
Density (kg/m^3)	10977	4448
Viscosity (Pas)		1E+21
Young's modulus (Pa)		3.9894E+11
Poisson's ratio		**0.49999**

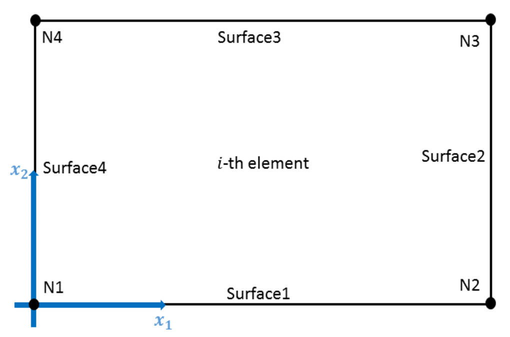

Figure1 Element configuration. x_1 and x_2 are local coordinates

Type of loading: Single degree harmonic load with 1) Heaviside loading history & 2) Saw tooth loading history is applied on the surface of the Earth

Element Type: ABAQUS CAX4H 4-node asymmetric bilinear element (fig.1)

Benchmark model

Analytical solution of Green's function for an incompressible, self-gravitating spherical shell is given by Sabadini & Vermeersen (2004) and Wu & Peltier (1984), where the Cauchy Residue Theorem is used during inverse Laplace transform (Wu 1978). Love number used in benchmark models are calculated by convolution between the Green's function and loading history.

Acknowledge

Computation were carried out in HPC2015, High Performance Computing Linux cluster system in The University of Hong Kong.

References

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Proposed Numerical scheme

Principle

Evaluate body force terms by iterations, in which starting solution is obtained by Wu (2004)'s stress transformation method.

Implementation of body force in ABAQUS

- Given radial displacement u_r, potential perturbation ϕ_1 is calculated by analytical solution of equation (2) (Eq.16 of Wu, 2004)
- Total force acting on an element is calculated by integrating body force throughout the whole volume of that element. In case of asymmetric geometry,

$$\boldsymbol{F_{total}} = \iint \nabla f \, dA = \oint f \, [\widehat{\boldsymbol{e_1}} + \widehat{\boldsymbol{e_2}}] \cdot \widehat{\boldsymbol{n}} \, ds \qquad (3)$$

where $f = -\rho_0 g_0 u_r - \rho_0 \phi_1$; $\widehat{\boldsymbol{e_1}}$ and $\widehat{\boldsymbol{e_2}}$ are unit vectors in local coordinates (fig.1) ; $\widehat{\boldsymbol{n}}$ is unit normal vector; closed path is taken along elements boundaries. The 2D form of Divergence Theorem (Cassel, 2013) is used here and total force acting on each element is applied as distributed load on element surface.

Two iteration schemes are proposed and tested:

Iteration scheme 1

For iteration 0 (same as Wu 2004)

- Self-gravity is not considered ($\phi_1 = 0$)
- Stress transformation method (Wu 2004) is used to obtain non-self-gravitating solution of Eq. (1) as starting solution
- All time steps $(t_1, t_2, \ldots, t_N)$ will be completed in one iteration

For iteration $k > 0$ (no stress transform)

- All time steps $(t_1, t_2, \ldots, t_N)$ will be completed in each iteration
- Self-gravity is considered, Eq.(1) will be solved
- Body force at each time step are calculated from displacement obtained in iteration $k-1$ via Eq.(3)

Iteration scheme 2

For iteration 0

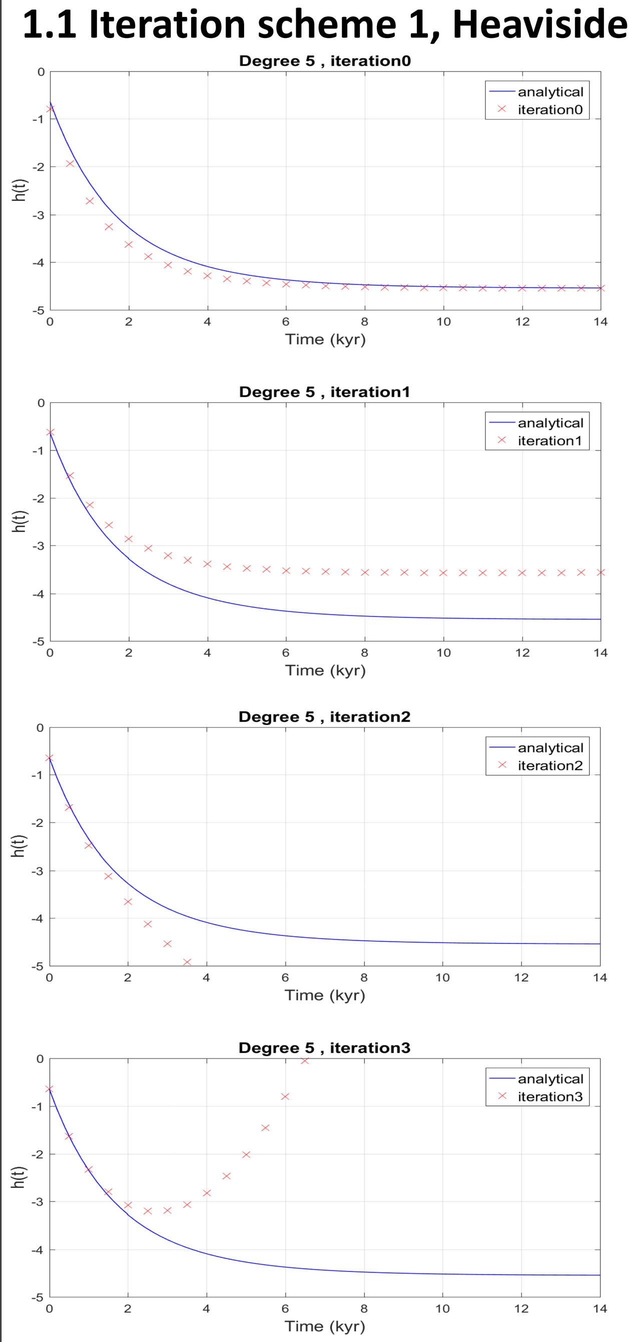
- Same as scheme 1

For iteration k $(0 < k \leq N)$ (no stress transform)

- Only the first k-*th* time steps will be done, i.e. $t_1, t_2, \ldots, t_k$
- Self-gravity is considered, Eq.(1) will be solved
- Body force at $t_1, t_2, \ldots, \boldsymbol{t_{k-1}}$ is calculated from displacement obtained in iteration $k-1$ via Eq.(3)
- Body force at $\boldsymbol{t_k}$ is calculated from displacement of iteration 0 via Eq.(3)

Results

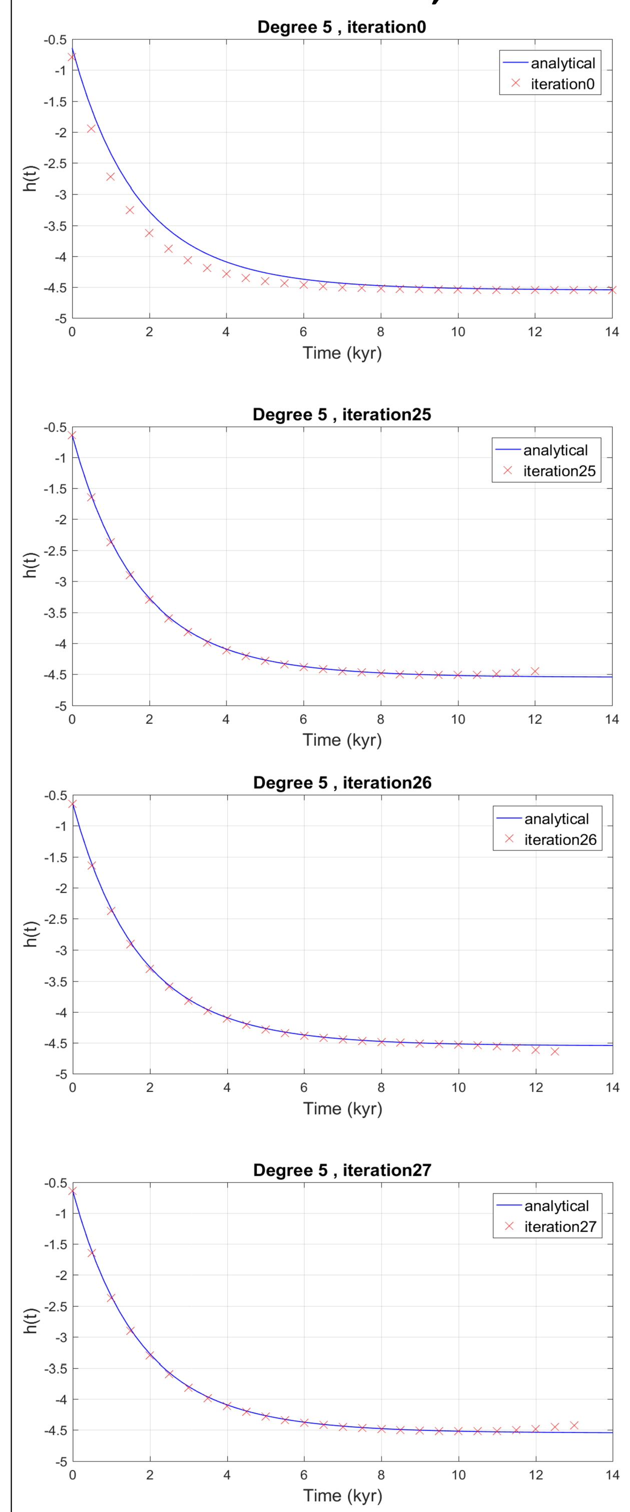
1.1 Iteration scheme 1, Heaviside loading, time increment = 0.5 kyr



- Iteration0 is the non-self-gravitating solution obtained by stress transformation (Wu, 2004), which is reasonably close to the analytical solution of Eq.(1)
- The viscous asymptote of iteration 0 already matches analytical solution
- Iteration0 is used to calculate initial guess of body force in subsequent iterations.
- Result failed to converge to the desired solution except at time step 1-3
- Similar pattern of divergence is observed in all tested time increment (1, 0.3 & 0.5 kyr)

Figure 2. h Love number against time, induced by degree 5 loading. Solid line is analytical solution obtained by normal model method. Markers are results obtained by iteration scheme 1. Only iteration 0 – 3 is shown, because the results are clearly diverging.

2.1 Iteration scheme 2, Heaviside loading, time increment = 0.5 kyr



- Iteration 0 is the non-self-gravitating solution obtained by stress transformation (Wu, 2004)
- Iteration0 is used to calculate initial guess of body force in subsequent iterations
- h Love number of first 10kyr matches pretty well with analytical solution
- The diverging behavior is significantly improved compare with scheme 1, but result still failed to converge after 10kyr
- Using the same time increment (0.5kyr), results of all tested degree of loading (2-10) failed to converge at different time, e.g. 1.5kyr for degree 2 & 14.5kyr for degree10

Figure 3. h Love number against time, induced by degree 5 loading. Solid line is analytical solution obtained by normal model method. Markers are results obtained by iteration scheme 2. Result of iteration0, 25, 26 & 27 are shown.

2.2 Iteration scheme 2, Heaviside loading, time increment = 0.3 kyr

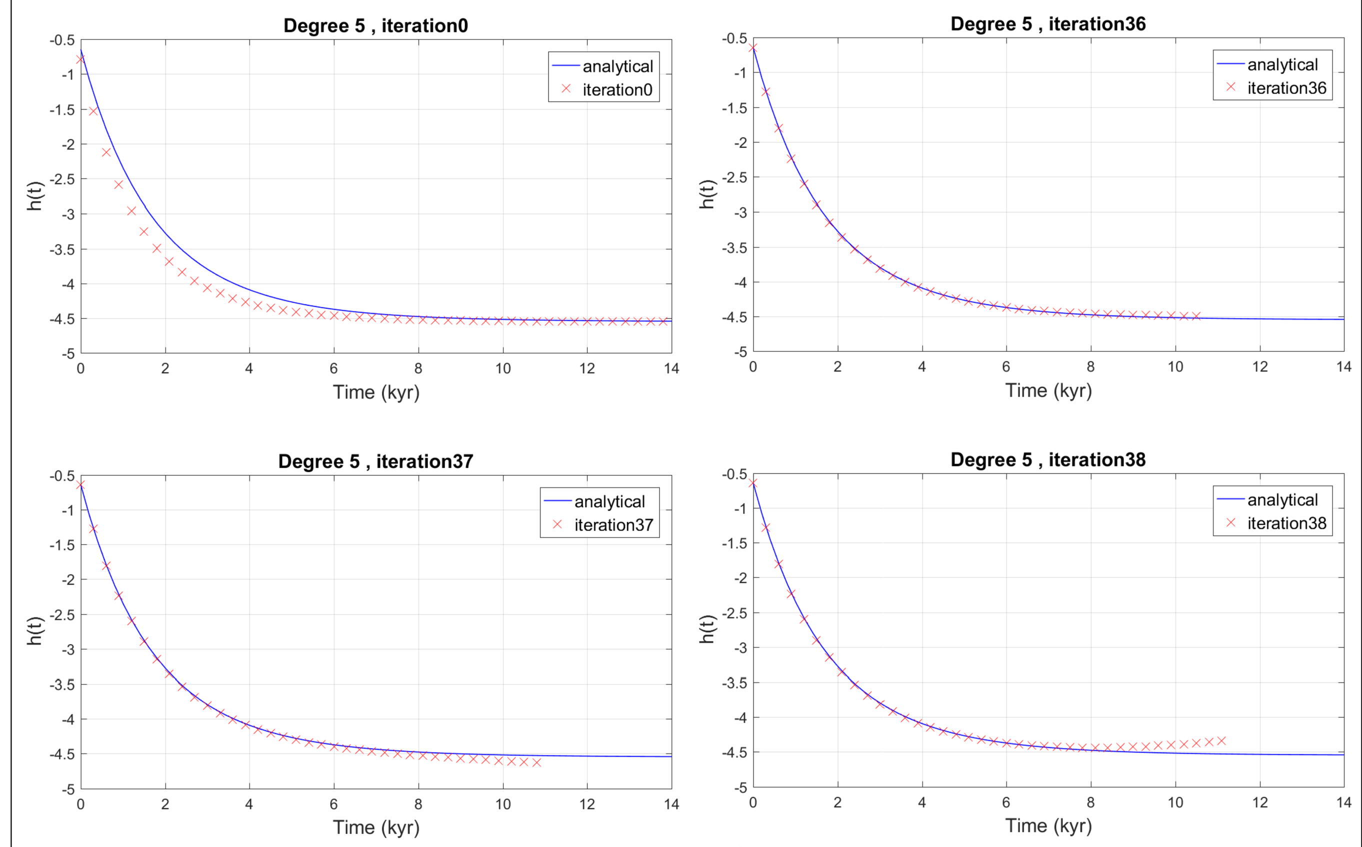


Figure 4. h Love number against time, induced by degree 5 loading. Solid line is analytical solution obtained by normal model method. Markers are results obtained by iteration scheme 2. Result of iteration0, 36, 37 and 38 are shown.

- Shorter time increment (0.3kyr) is used
- Only result of the first 8 kyr match with analytical solution, which is even worse than the result obtained by using time increment 0.5 kyr
- For degree 5 loading, result failed to converge after 8 kyr

2.3 Iteration scheme 2, Saw Tooth loading, time increment = 0.5 kyr

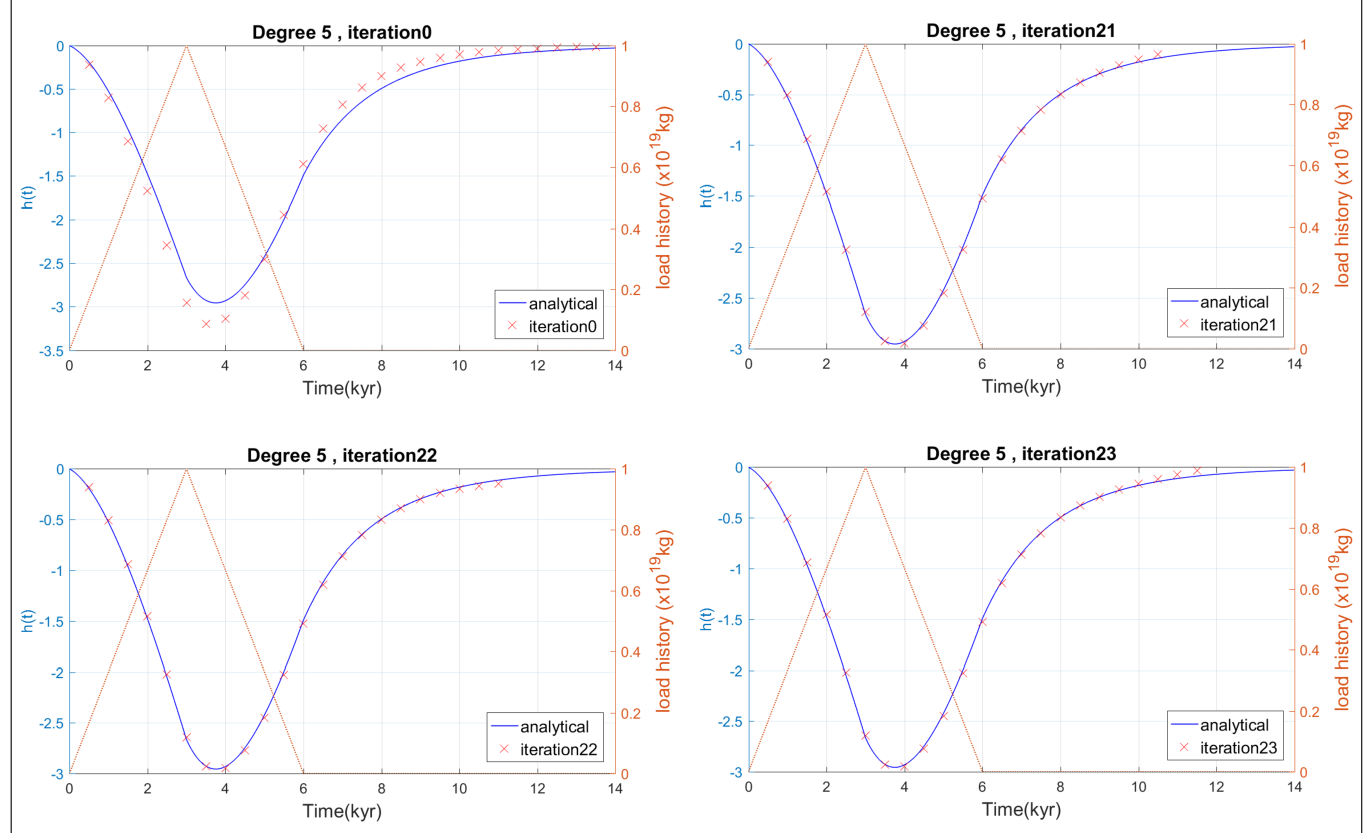


Figure 5. h Love number against time, induced by degree 5 loading. Solid line is analytical solution obtained by normal model method. Markers are results obtained by iteration scheme 2. Loading history is indicated by dotted orange line, where loading phase is 0-3kyr and unloading phase is 3-6kyr

- Again, iteration 0 is the non-self-gravitating solution obtained by stress transformation (Wu, 2004)
- h Love number agrees with analytical solution for the first 10 kyr, after that, result failed to converge
- Results still diverge even when loading is removed

Conclusion

1. Results obtained by iteration scheme 2 is much better than scheme 1
2. The diverging behavior could not be improved even by shortening time increment of finite element analysis
3. Even when loading reduced to zero, results still failed to converge at large time
4. We suspect that failure of convergence may be due to numerical errors accumulated during successive iterations.