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Hamiltonian Formulation of the Rotating Shallow Water Equations using Split Exterior Calculus

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We present a novel formulation of the rotating shallow water equations in Hamiltonian form, using twisted differential forms. This is a natural extension of [1], and provides a clean basis for the topological - metric split employed in that paper: the Hamiltonian encodes the metric information, while the Poisson brackets are purely topological. Using this new continuous formulation, a general discrete exterior calculus based numerical scheme is developed. It is shown that the TRiSK family of schemes (cf. [3,4]) is one particular example of the general scheme. This completes the characterization of TRiSK as a DEC scheme started in [5] and further developed in [2], by providing an understanding of all the operators that appear in terms of discrete versions of fundamental exterior calculus operators: the Hodge star, the wedge product and the exterior derivative. It is believed that this new understanding of TRiSK as a complete DEC scheme will provide a pathway to overcoming the accuracy issues of TRiSK on quasi-uniform spherical grids in a way that does not destroy its key properties. This is the subject of future work.

References

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