



An assessment of Peters-Durner-Iden (PDI) hydraulic conductivity function: Enhancing prediction of unsaturated hydraulic conductivity by estimating parameters within a Bayesian framework

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Hydraulic conductivity (HC) of porous media is proportional to the volumetric water content or the capillary head, and proper estimation of hydraulic conductivity curve (HCC) is mandatory for modeling liquid movement through porous media. It can be challenging to measure unsaturated HC and measurement apparatuses are time-consuming and costly.

One of the recently developed hydraulic conductivity functions (HCFs) based on the concept of a bundle of capillary tubes is Peters-Durner-Iden (PDI) model which has found great success in modeling two-phase flow. The PDI model is comprised of a liquid conductivity and an isothermal vapor conductivity terms, where liquid conductivity itself is expressed by the summation of capillary and film conductivity terms. In this research, van Genuchten model was used to describe capillary term. The procedure commonly carried out to determine PDI model consists of using a nonlinear regression algorithm which minimizes the sum of weighted squared residuals between model prediction and data pairs. But at first, the four water retention function (WRC) parameters (i.e. volumetric residual water content, volumetric residual water content and two shape parameters of van Genuchten model) are determined through a fitting procedure. Once water retention parameters are known, four HC parameters (i.e. Capillary saturated HC, film saturated HC, pore tortuosity parameter and an empirical film flow parameter) are estimated by fitting HCF to measured data. In this research, a Bayesian framework is used to minimize a coupled objective function comprised of a summation of water retention and HC objective functions. The commonly used local optimization methods are likely to get trapped in local minima, especially when dealing with high dimensional problems. Hence, Differential Evolution Markov Chain Monte Carlo (DE-MCMC) that employs multiple starting points having the highest likelihood for a Markov chain in each run was used to find an estimate of the global optimum. The results indicate that in most cases DE-MCMC provides a more accurate estimation of PDI parameters according to both Nash-Sutcliffe Efficiency (NSE) and Bayesian Information Criterion (BIC).

Despite presenting a very accurate fit, PDI model suffers from a major drawback of requiring at least one measured HC at the dry ranges of saturation and measurement of HC at low water contents becomes extremely time-consuming due to disconnection of water filled paths. That is because the parameters of film conductivity model presented in PDI cannot be inferred from WRC directly. The excessive amount of time required to reach equilibrium state is the main issue that inspired many researchers to present HCFs that require no prior knowledge of HCC. Therefore, PDI model can be used to interpolate missing hydraulic conductivities and with the presented Bayesian framework the uncertainty of model parameters associated with the available amount of observed HC can be presented as well.