



Stochastic partial differential fluid equations as a diffusive limit of deterministic Lagrangian multi-time dynamics

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In {Holm, Proc. Roy. Soc. A 471 (2015)} stochastic fluid equations were derived by employing a variational principle with an assumed stochastic Lagrangian particle dynamics. Here we show that the same stochastic Lagrangian dynamics naturally arises in a multi-scale decomposition of the deterministic Lagrangian flow map into a slow large-scale mean and a rapidly fluctuating small scale map. We employ homogenization theory to derive effective slow stochastic particle dynamics for the resolved mean part, thereby justifying stochastic fluid partial equations in the Eulerian formulation.

To justify the application of rigorous homogenization theory, we assume mildly chaotic fast small-scale dynamics, as well as a centering condition. The latter requires that the mean of the fluctuating deviations is small, when pulled back to the mean flow.