



Elliptical instability of inertial waves in rotating fluids: Hamiltonian formalism and numerical simulation

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Elliptical instability in rotating fluids has been studied for almost fifty years both theoretically and experimentally [1]. The ellipticity of the flow plays the role of parametric modulation and leads to the excitation of pairs of inertial waves. The linear stage of elliptical instability is well known, while its nonlinear regime raises many important questions [2]. The rotation of the Earth together with tidal deformations make the study of elliptical instability is among the most important geophysical problems. Recently we developed a Hamiltonian approach for inertial waves in rapidly rotating fluids [3]. Its applications to the circularly rotating flows was presented on the conference EGU 2017 [4]. In this work we extend the Hamiltonian formalism to the case of weakly elliptic flows. We present results of theoretical and numerical studies for the linear and nonlinear stage of elliptical instability. In the first case we find expressions for the threshold value of the ellipticity for the viscous fluid in a rotating tank of a finite size when the instability starts to grow. This theoretical estimation is in good agreement with direct numerical simulations performed using the open source Snoopy code [5], developed by Lesur [6] for circular flows and later adapted to study elliptical case by Barker [7]. For the nonlinear stage of elliptical instability, we obtain a theoretical estimation for the so-called phase mechanism of the amplitude saturation. Finally, we discuss this mechanism using the results of the numerical simulations. The work is supported by the RFBR (Grant No.16-31-60086 mol_a_dk). We are grateful to the Authors of the paper [2] for providing us an updated version of the Snoopy code and useful comments regarding numerical simulation of the elliptical instability.

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