



Localized Radial Basis Functions' (LRBF) solution of the two-dimensional Fokker-Planck equation

Seyyed Mahmoud Mousavi (1) and Christina W. Tsai (2)

(1) National Taiwan University, Civil Engineering, Taiwan (musavi6889@yahoo.com), (2) National Taiwan University, Civil Engineering, Taiwan (christina_w_tsai@yahoo.com)

The Fokker-Planck equation is a partial differential equation (PDE) that can be used to illustrate the stochastic systems by a probability density function. The Fokker-Planck equation can describe the stochastic phenomenon of the random motions of finer particles in a fluid, e.g. suspended sediment. In fact, the Fokker-Planck equation is the counterpart of the advection-diffusion (A-D) equation in deterministic models and plays an important role in stochastic transport problems. This PDE aims at quantifying temporally and spatially varying concentrations. In this study, we discretize the temporal variable with the second-order Adams-Bashforth method. The multiquadric (MQ) is applied to the spatial variable. The MQ is one of the famous Radial Basis Functions (RBFs). The method of Radial Basis Function is intrinsically a meshless method. It is aptly named, as merely a set of uniform or non-uniform nodes should be scattered in the domain of interest without knowing any node-to-node connectivity a priori. There are two well-acknowledged difficulties with the RBFs. Firstly, most of these basis functions depend on a constant (shape) parameter, which is not easy to be suitably determined. Secondly, these RBFs lead to a system of algebraic equations, i.e. $Ax=b$, where the matrix A is a full and ill-conditioned matrix. We utilize the localization technique (LMQRBF) to resolve the two aforementioned drawbacks. The potential usefulness of the localized RBFs for the Fokker-Planck equation can be demonstrated in this study.

Keywords: Fokker-Planck equation, Meshless, Multiquadric, Localization, Suspended sediment, Stochastic system