



The onset of chaos in nonautonomous dissipative dynamical systems: A low-order ocean–model case study

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The transition to chaos induced by periodic forcing in systems that have regular behavior in the autonomous limit is studied with a four-dimensional nonlinear spectral ocean model. The analysis is based on the systematic construction of the system's pullback attractors (PBAs) through ensemble simulations derived from a large number of initial states in the remote past.

We carry out a preliminary analysis of the autonomous system by constructing its bifurcation diagram, and we calculate a metric that measures the mean distance between two initially nearby trajectories, as well as the system's entropy. Regular, nonchaotic attractors are found to exhibit sensitive dependence on initial data; this apparent paradox is resolved by noting that the dependence only concerns the phase of the periodic trajectories, and it disappears once the latter have converged onto the attractor.

The periodically forced system yields periodic or chaotic PBAs depending on the periodic forcing's amplitude. A new diagnostic method — based on the cross-correlation between two initially nearby trajectories — is proposed to characterize the transition between the two types of behavior. Transition to chaos is found to occur abruptly at a critical value, and begins with the intermittent emergence of periodic oscillations with distinct phases. Furthermore, a standard Van der Pol oscillator is used to help explain the co-existence in the phase space of a quasi-periodic PBA and a chaotic PBA, for periodically forced systems that are close to a hamiltonian skeleton; this simple example shares key dynamical features with our low-order ocean model.