



Drucker-Prager visco-plasticity does not converge. But visco-elasto-plasticity does.

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Dynamic pressure-dependent Drucker-Prager visco-plastic rheology is commonly used in numerical geodynamic modeling of lithospheric processes, which show prominent spontaneous strain localization phenomena resulting in formation of various patterns of shear zones. These shear zones are typically one-two grid step wide and the pattern is typically non-unique and changes with changing model resolution (e.g., Buiter et al., 2006). On the other hand, shear zone angles and some other characteristics (e.g., taper angle, gross-scale strain pattern etc.) seem to be relatively robust and agree with both theoretical predictions and analogue models (e.g., Buiter et al., 2006, 2016). Recently, it has been demonstrated (Spiegelman et al., 2016) that problems arise with the finite-element-based numerical solvability of the incompressible Stokes problems for rheologies that depend on the dynamic pressure such as Drucker-Prager visco-plasticity. Analysis suggests that in this case visco-plastic incompressible Stokes problems can become ill-posed and do not converge in term of accurately reproducing the yielding condition on the grid. Here I report similar problem for staggered finite difference marker in cell approach and offer the way to overcome it by using visco-elasto-plastic (rather than visco-plastic) rheological model with an adaptive time stepping. Test calculations show that in case of 2D staggered grid, several rules have to be followed to achieve a computer accuracy solution: (1) Drucker-Prager yielding conditions should be only controlled in the vertex points (where shear stresses are defined), which serve as master nodes for adjusting the effective visco-plastic viscosity, (2) this viscosity should be then interpolated to the slave pressure points (where normal stresses are defined) by using harmonic (rather than arithmetic or geometric) average and (3) in case of non-convergence, the size of the time step should be reduced and iterations should be restarted from the beginning (i.e. from the previous physically meaningful state of stresses and visco-plastic viscosities on the grid). I also show that this iterative solution method could be extended to fully coupled hydro-mechanical visco-elasto-plastic problems, in which the effective pressure (i.e. the difference between the total and fluid pressure) is used in the yielding condition instead of the dynamic pressure.

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Spiegelman, M., D. A. May, and C. R. Wilson (2016), On the solvability of incompressible Stokes with viscoplastic rheologies in geodynamics, *Geochem. Geophys. Geosyst.*, 17, 2213–2238,