Geophysical Research Abstracts
Vol. 20, EGU2018-13067-1, 2018
EGU General Assembly 2018

Equations for 1D waves on the surface of deep water

Dmitry Kachulin (1), Alexander Dyachenko (1,2), Vladimir Zakharov (1,2,3)

(1) Novosibirsk State University, Russian Federation, (2) Landau Institute for Theoretical Physics RAS, Chernogolovka, Russia, (3) University of Arizona, Tucson, USA

We apply a canonical transformation to a water wave equations to remove cubic nonlinear terms and to drastically simplify fourth-order terms in the Hamiltonian. This transformation from natural Hamiltonian variables η, ψ to new complex normal variables c, c^* explicitly uses the fact of vanishing exact four-wave interaction for water gravity waves for a 2D potential fluid. The new variable is the sum $c(x,t) = c^+(x,t) + c^-(x,t)$ of two analytic functions: $c^+(x,t)$ – is analytic in the upper half-plane, $c^-(x,t)$ – is analytic in the lower-plane. We obtained system of two coupled differential equations for c^+ and c^- which is very suitable for analytical studies and numerical simulations:

$$
\begin{aligned}
\frac{\partial c^+}{\partial t} + i\hat{\omega} c^+ &= \partial_x^+ \left[i\left(|c^+|^2 - |c^-|^2\right) c_x^+ + c^+ \hat{k}\left(|c^+|^2 - |c^-|^2\right) - i c^+ c^- {c_x^-}^* - {c^-}^* \hat{k}\left(c^+ c^-\right) \right], \\
\frac{\partial c^-}{\partial t} + i\hat{\omega} c^- &= \partial_x^- \left[i\left(|c^-|^2 - |c^+|^2\right) c_x^- - c^- \hat{k}\left(|c^-|^2 - |c^+|^2\right) - i c^- c^+ {c_x^+}^* + {c^+}^* \hat{k}\left(c^+ c^-\right) \right]
\end{aligned}
\qquad (1)
$$

Here $\hat{\omega}$ and $\hat{k}$ are correspond to the multiplication by $\sqrt{gk}$ and $|k|$ in the Fourier space, $*$ denotes complex conjugation, the subscript x is the derivative with respect to the variable x, the differentiation operators ∂_x^+ and ∂_x^- are $ik\Theta(k)$ and $ik\Theta(-k)$, where $\Theta(k)$ is the Heaviside step function. Physical variables $\eta(x,t)$ and $\psi(x,t)$ can be restored from complex variable $c(x,t)$. The system (1) has the simple solution:

$$
c^+ = A e^{ik_A x - i\omega_A t}, \qquad c^- = B e^{-ik_B x - i\omega_B t},
$$

where

$$
\begin{aligned}
\omega_A &= \omega_{k_A} + k_A^2(|A|^2 - |B|^2) - k_A k_B |B|^2 + k_A |k_A - k_B| |B|^2 \\
\omega_B &= \omega_{k_B} + k_B^2(|B|^2 - |A|^2) - k_A k_B |A|^2 + k_B |k_A - k_B| |A|^2
\end{aligned}
$$

We performed numerical simulation of system (1) for water waves moving in opposite directions.