



Historical trends, key figures and tipping points in the mathematics of free-surface flow and erosion

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Free-surface flow models coupled (or not) to erosion models cover a wide range of applications and spatiotemporal scales, from the infra-plot scale to large river basin scales, and from the second to the year. This work handles the historical aspects, multidisciplinary paths and main chronological trends in the state of the art up to now in free-surface flow (hydrological and hydraulic) modelling with extensions towards friction laws and erosion issues. Special emphasis is added to the help provided by mathematical tools, numerical methods and computer science in the most recent times. The methodology is based on identifying the main trends and tipping points between disciplines using a "tree timeline". Three time periods were separately analyzed, corresponding to Antiquity-Renaissance, Renaissance-1950's and 1950's-nowadays. We discuss the chronological structure and relative position of major findings or beliefs for each time period, then propose Key Figures for each discipline, leading to the identification of main trends and tipping points (e.g. breakthroughs or paradigm shifts).

The Antiquity-Renaissance period is mostly characterized by evolutions that took place from the Dark Ages to the beginning of the 17th century, from mystical to natural hydrology, from hydrology to hydraulics, from isolated developments to their tentative assembly within mathematics formalism, and from the early Greek philosophies of Cosmology to first tokens of observation-based erosion science.

The Renaissance-1950's period is when observations gave rise to theories, delineating two major trends that originate in the contributions of the Geniuses of the Renaissance, especially Leibniz and Newton. The development of mathematical descriptions of flows may be seen as "Leibniz's wake", finally resulting in the derivation of the four systems of differential equations that provide most theoretical material for free-surface flow studies: Navier-Stokes, Reynolds-Averaged-Navier-Stokes, Saint-Venant and Approximations to Saint-Venant (or equivalent formulations), listed here by decreasing refinement. Meanwhile, Newton's wake, maybe more on the physics side, saw the development of empirical friction laws, then the apparition of erosion equations, possibly seen as distant consequences of his early derivation of the laws of (fluid) motion.

The 1950's-nowadays period, corresponding to recent research in hydrological, hydraulic and erosion modelling, has inherited from both the former mathematical and physical backgrounds, combining mechanistic and empirical equations, and feeding holistic approaches with contextual data. In recent years, all forms of Environmental Fluid Mechanics (either in simplified or refined formalisms) have greatly benefited from advances in numerical methods and computer science, clearly turning towards transdisciplinarity in the Earth Sciences.