

Solitary wave collisions in the integrable NLS model and in the compact Zakharov equation for envelope

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In the complex dynamics of nonlinear waves on the surface of deep water, one type of processes focus special attention of experimental and theoretical studies: the propagation of coherent wave structures – solitons and breathers and their interactions. The Euler equations for deep water being exact model for the surface waves are rather complicated for analytical analysis as well as for numerical simulations. For this reason, the study of approximate models for the deep water surface waves plays important role in fluid dynamics and geophysics.

The first order weakly nonlinear model for the surface waves is the nonlinear Schrodinger (NLS) equation. The interactions of solitons in the frame of integrable model NLS are elastic – i.e. solitons restore their shape after collisions. Moreover the powerful approach of the inverse scattering transform allows to describe soliton interactions via exact N -soliton solution formulas. In nonintegrable models such exact solutions cannot be found. However, numerical simulations for different nonintegrable models, e.g. compact Zakharov equation also demonstrate the presence of stable coherent structures – breathers (or solitons). We study compact Zakharov equation in the form suggested in the work [1], that describe the wave train envelope (but without any assumptions about its spectral width). The envelope form of the compact Zakharov equation allows us to make straightforward comparison with less accurate (but integrable) NLS model. We focus of the two aspects of solitons collision process: solitons phase synchronization and excitation of radiation (inelasticity of the interaction).

The interactions of solitons in compact Zakharov equation lead to excitation of minor radiation [2]. Nevertheless, we demonstrate that the general properties of the collision process are similar to the case of the NLS equation. This similarity is observed even at high wavefield steepness. In particular, soliton interactions can lead to the formation of extreme amplitude waves at certain phase synchronization. It is well known, that maximum value of amplitude amplification as a result of soliton interactions in the NLS model is equal to the sum of soliton amplitudes $C_1 + C_2$. We found, that in the compact Zakharov equation the maximum amplification can be significantly higher then $C_1 + C_2$ at large values of the wave steepness. Finally we show how the level of soliton radiation depends on the relative phase of the colliding solitons, that extends results of the work [2].

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References

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