

General dispersion relation in non-linear seismic metamaterials modelled as periodic mass-spring systems

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We study the dynamical properties of one-dimensional non-linear seismic metamaterials modelled as periodic arrangements of mass-spring systems. This is accomplished by including higher-order anharmonic interactions modelled by an anharmonic potential. The potential energy is expanded to all orders setting equal to zero odd orders that would lead to a nonreciprocal behavior of the wave propagation inside the metamaterial. The non-linear equation of motion is written in terms of an anharmonic force and its general solution is expressed by the rectangular function. The general dispersion relation is obtained applying the Fourier transform method to the rectangular function that can be projected onto the local wave propagating inside the metamaterial. We have numerically investigated the realistic case including a third-order force and the dynamics described by a Duffing equation. Because of the nonlinearity, the acoustic branch becomes evanescent in a given range of wave vectors. The condition of the wave amplitude assuring the suppression of the acoustic wave frequency is discussed for different values of the wave amplitude. This model is extremely useful and can be applied to conduct accurate seismic response analyses of horizontally layered soil deposits.