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Modification of Zakharov equation for intermediate water depth

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The Hamiltonian properties introduced by Zakharov [6] stimulated the author to apply successive canonical transformations which isolate the skeleton of the important dynamics. Interactions that can not be removed are the ones that truly participate to the dynamics (resonant modes). The other ones can be considered as simple shape correction (bound modes). One can solve the condensed initial value problem for reduced canonical variable and observe the evolution of the important dynamics, for example the effects on waves propagation speed, or the strength of energy exchange among components. Whenever it is necessary to come back to "reality", the reduced system solution has to be transformed back to physical variables.
It is now widely accepted that, discarding surface tension, quadratic interactions do not participate to the dynamics. This fact has been verified for deep water regimes. But, even if no triad resonance is possible in finite water depth, the role of a mean flux still remains a conundrum [1, 2]. One of the consequences is that it is not possible to deduce the correction to the linear dispersion relation due to cubic terms [5] as it can be done in 1D cases [3].
We show that the reduced canonical variable equations cannot be integrated on a simple lattice with the usual techniques, even if some special cases exist. Moreover, the direct and inverse transformations linking the "reduced" and the "physical" actions are singular. The reasons proceed from the fact that even if the involved integrals may exist (in a improper sense) some of the kernels are not defined on the trivial resonance manifold. For example, using Krasitskii [4] notation, the limit $\widetilde{V}^{(2)}_{1,2,1,2}$ does not exist [7]. Wishing to maintain the overall solution method, we find that smooth kernels can be obtained only relaxing the Zakharov canonical transformation, that is allowing some quadratic interactions in the dynamics. The impacts of this finding will be discussed also comparing the predictions of the proposed equation, the Zakharov equation and a High-Order Spectral scheme.

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